Signals and Systems-23EC02

UNIT-I

Introduction

B.Tech., III-Sem., ECE

1. Definition and Classification of Signals

- Continuous Time or Analog Signals
- Discrete Time and Digital Signals

2. Definition and Classification of Systems

- Continuous Time Systems
- Discrete Time and Digital Systems

3. Singularity and Related Signals

- Impulse Signal
- Step Signal
- Signum Signal
- Ramp Signal

4. Complex Exponential and Sinusoidal Signals

5. Operations on Signals

- Time Shifting
- Time Scaling
- Time Reversal
- Amplitude Shifting
- Amplitude Scaling

6. Properties or Characteristics of Signals

- Even and Odd Signals
- Causal and Noncausal Signals
- Bounded and Unbounded Signals
- Periodic and Aperiodic Signals
- Deterministic and Random Signals
- Energy and Power Signals

7. Analogy between Vectors and Signals

8. Approximation of a Signal by another Signal

- Mean Square Error (MSE)
- Condition for Orthogonal Signals

9. Orthogonal Signal Space

10. Approximation of a Signal by set a of mutually orthogonal Signals

- Mean Square Error (MSE)
- Gibb's Phenomena

11. Orthogonality in complex Signals

- Approximation of a complex signal by another complex signal
- Approximation of a complex signal by a set of mutually orthogonal complex signals

12. Solved Problems

13. Assignment Questions

14. Quiz Questions

1. Definition and Classification of Signals:

Signal can be defined as a function or any physical phenomenon that conveys or carries some information and its amplitude may vary with respect to one or more independent variables.

Examples:

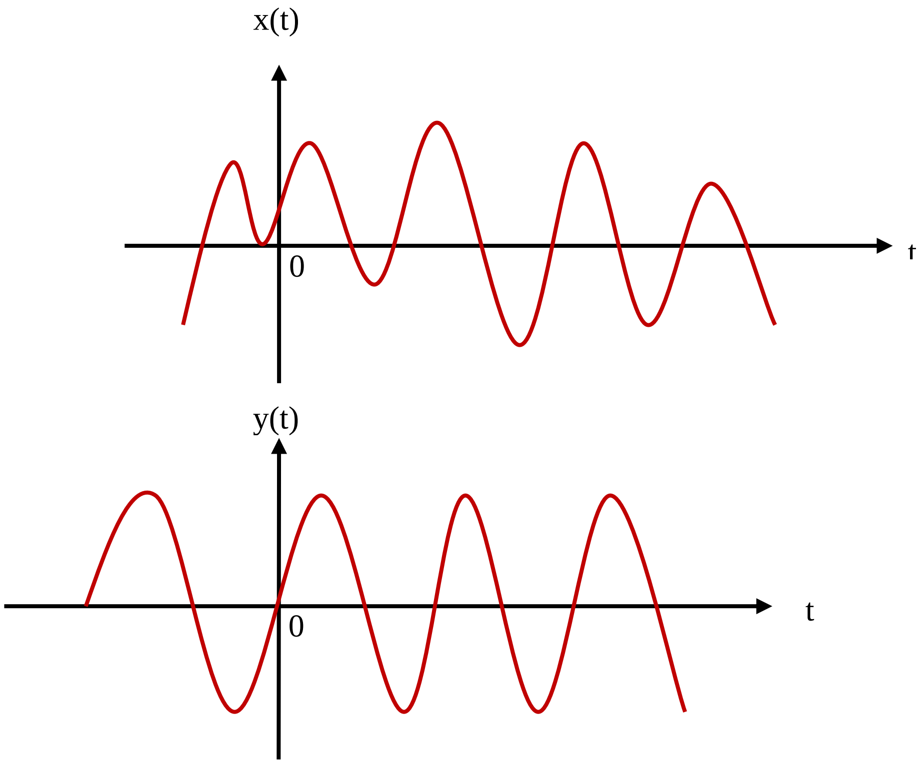
- Music Signal, Speech Signal, Video Signal
- Electrocardiogram (ECG) Signal – It is used to predict heart diseases
- Electroencephalogram (EEG) Signal – Study the normal and abnormal behavior of the brain
- Electromyography (EMG) – It is used to study the condition of muscles
- Electromagnetic Waves
- Radar Signals

Based on variation in amplitude, signals are classified into mainly two types.

- Continuous Time or Analog Signals
- Discrete Time and Digital Signals

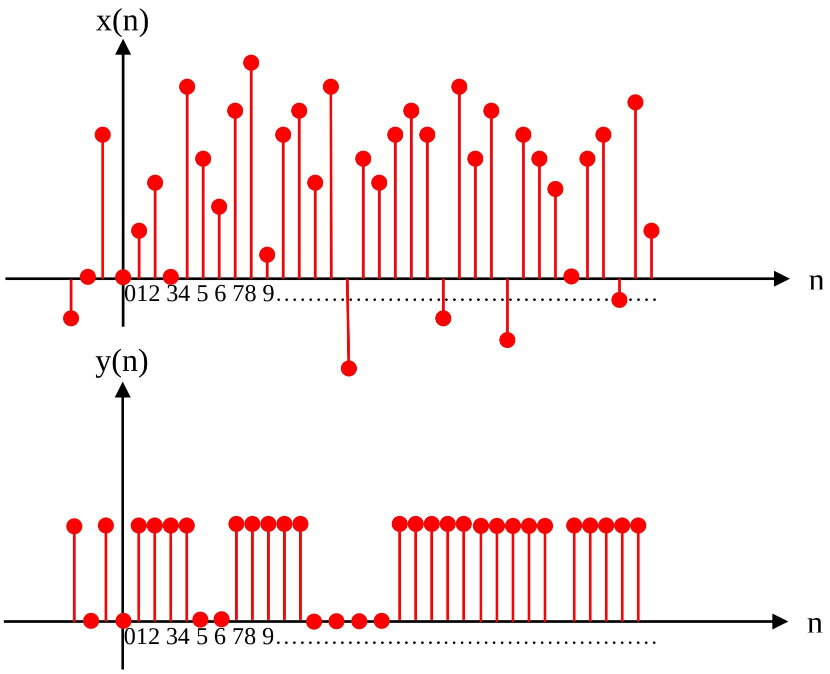
1.1. Continuous Time or Analog Signals:

Continuous time signals are those for which the amplitude varies continuously in accordance with continuous variation in time. All real time signals are analog in nature; hence the continuous time signals are also known as analog signals. In general, continuous time signals are represented with x(t), y(t), z(t), etc.



1.2. Discrete Time and Digital Signals:

Discrete time signals are those for which the amplitude varies discreetly in accordance with discrete variation in time. Any discrete time signal can be represented as the sequence of numbers, that's why discrete time signals are called sequences. Discrete time signals can be obtained from continuous time signals by sampling process(Sampling Theorem). In general, discrete time signals are represented with x(n), y(n), z(n), etc.



Amplitude restricted version of discrete time signals are called digital signals, for which the different number of amplitudes are restricted to finite number (Two). y(n) is a digital signal and all digital signals are discrete time signals. Digital signals can be obtained from discrete time signals by quantization mechanism.

Examples:

- $x(t) = 2Cos(3t) + 3Sin(2t)$
- $y(t) = 3e^{-2t}$
- $z(t) = 3e^{-2t}Cos(4t)$

} Continuous time or analog signals

- $x(n) = 2^n$
- $y(n) = 2Cos(3n - 4)$
- $z(n) = 2e^{jn\pi/3}$

} Discrete time signals

- $x(n) = \{1,1,0,1,0,1,1,1,0,0,1\}$
- $y(n) = \{1,-1,1,-1,1,1,-1,-1,1\}$
- $z(n) = \{1,1,2,1,2,1,1,1,2,2,1\}$

} Discrete time or Digital signals

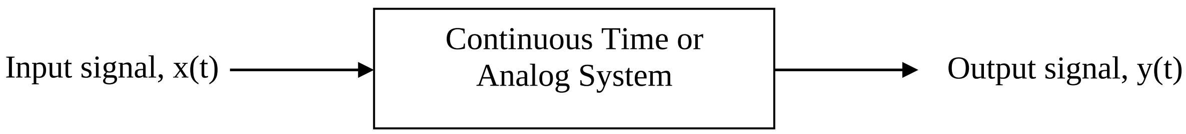
2. Definition and Classification of Systems:

System can be defined as the collection of objects or elements or components and all those things should be interconnected in such a way to achieve an objective or predefined result or outcome. Based on the type of input applied, components used in the design and type output, systems are classified into two types.

- Continuous Time or Analog Systems
- Discrete Time and Digital Systems

2.1. Continuous Time or Analog Systems:

Continuous time or analog systems are those for which both the input and output are continuous time signals and are constructed by using analog components, like resistors, capacitors, inductors, diodes, transistors, analog ICs, etc.



- Thermal stability of continuous time or analog systems is poor because of all analog components are temperature sensitive.
- Continuous time or analog systems are non-programmable and static in nature.
- Continuous time or analog systems are described by differential equation, which involves only differentials.

Examples:

(a) A simple RC high pass filter acts as a differentiator, where the output y(t) is the differentiation of input x(t)

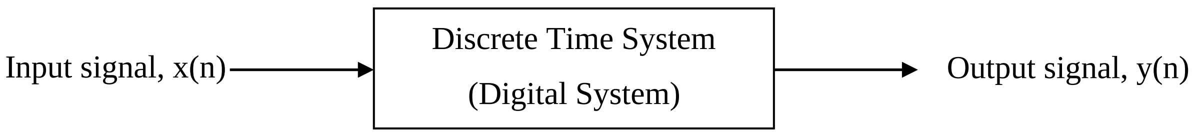
$$y(t) = \frac{d}{dt}\,[\,x(t)\,]$$

(b) A simple RC low pass filter acts as an integrator, where the output y(t) is the integration of input x(t)

$$y(t) = \int x(t)dt \Rightarrow x(t) = \frac{d}{dt}[y(t)]$$

2.2. Discrete Time and Digital Systems:

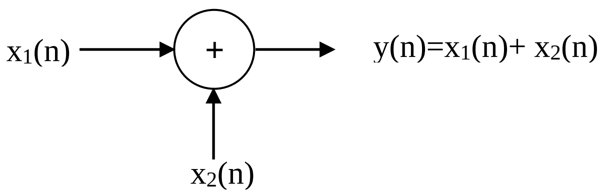
Discrete time systems are those for which both the input and output are discrete time signals. In the case of digital systems, both input and output are digital signals.



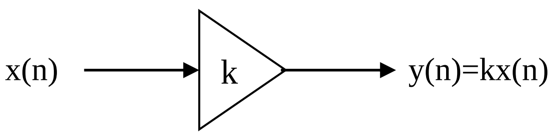
- Thermal stability of discrete time or digital system is high.
- Discrete time or digital systems are programmable and dynamic.
- Discrete time or digital systems are described by a difference equation, which does not involve differentials, which involve only shifts.

Discrete time or Digital systems are constructed by using discrete components, like adders, constant multipliers and delays (memories).

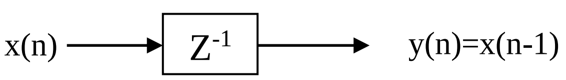
Adder is used to add two or more signals, for example $y(n) = x_1(n) + x_2(n)$



Constant multiplier is used to get the product of a constant and a signal, for example $y(n)=kx(n)$



Delay or Memory unit is used to get one unit delay,



Example, $y(n) = 2x(n)+3y(n-1)$

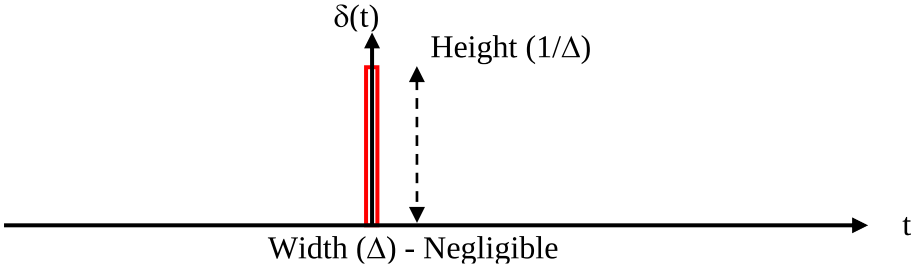
3. Singularity and Related Signals:

Singularity signals are those for which the left and right limits are not same, i.e. Singularity signals are discontinuous at a particular point or points, like, Impulse signal, Step signal, Signum signal and Ramp signals.

3.1. Impulse Signal:

Impulse signal is denoted with δ(t) and it is also known as Dirac delta signal

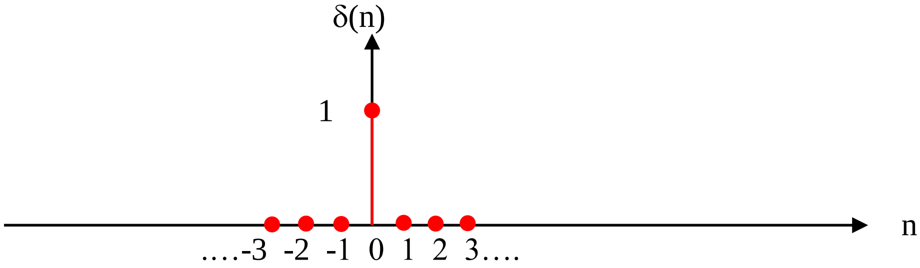
$$\delta(t) = \begin{cases} \infty & ; \ t = 0 \\ 0 & ; \ t \neq 0 \end{cases}$$



- Impulse signal δ(t) is even
- $\frac{d}{dt}\delta(t)$ is a doublet signal and it is odd.
- Area under any impulse signal is '1' $\Rightarrow \int_{-\infty}^{\infty} \delta(t)dt = 1$
- $\delta(at) = \frac{1}{|a|}\delta(t)$, $a \neq 0$.
- $x(t)\delta(t-t_1) = x(t_1)\delta(t-t_1)$ and $\int_{-\infty}^{\infty} x(t)\delta(t-t_1)dt = x(t_1)$
- $\int_{-\infty}^{\infty} x(t)\frac{d^n}{dt^n}\delta(t-t_1)dt = (-1)^n \frac{d^n}{dt^n}x(t) \ \ at\ t = t_1$

Note: Impulse signal in discrete domain is called **Digital impulse or unit sample sequence** and it is denoted with δ(n) and it can be defined as

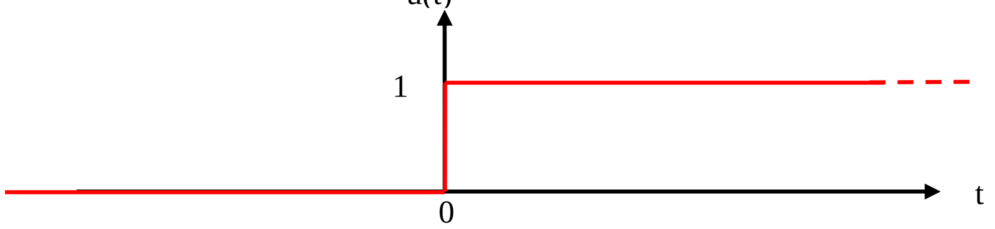
$$\delta(n) = \begin{cases} 1 & ; \ n = 0 \\ 0 & ; \ n \neq 0 \end{cases}$$



3.2. Step Signal:

Step signal is denoted with u(t) and it is also known as Heaviside step signal.

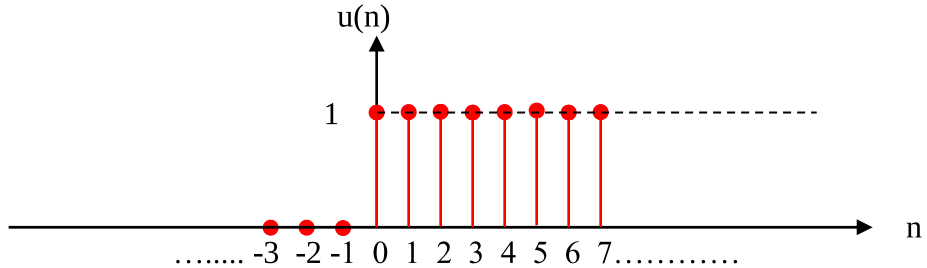
$$u(t) = \begin{cases} 1 & ; \ t > 0 \\ 0 & ; \ t < 0 \end{cases}$$



- It is neither even nor odd signal
- It is discontinuous at t = 0 ⇒ $\underset{t \to 0^-}{\text{Lt}} u(t) \neq \underset{t \to 0^+}{\text{Lt}} u(t)$
- Relation between u(t) and δ(t) is $\delta(t) = \frac{d}{dt}u(t)$ and $\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$

Note: Step signal in discrete domain is denoted with u(n) and it can be defined as

$$u(n) = \begin{cases} 1 & ; \ n \geq 0 \\ 0 & ; \ n < 0 \end{cases}$$



Step signal u(n) is the sum of a train of unit sample sequences

$$u(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \ldots\ldots\ldots + \delta(n-k) + \ldots\ldots$$

$$u(n) = \sum_{k=0}^{\infty} \delta(n-k) \quad \text{or} \quad u(n) = \sum_{k=-\infty}^{n} \delta(k)$$

Unit sample sequence δ(n) is difference between u(n) and u(n-1)

$$\delta(n) = u(n) - u(n-1)$$

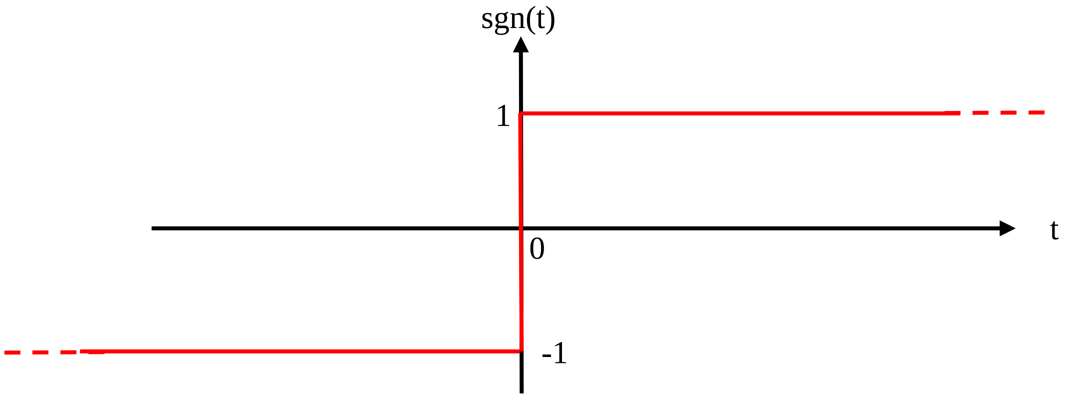
A sequence x(n) can be represented using unit sample sequence δ(n)

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

3.3. Signum Signal:

It is a double step signal and it is denoted with sgn(t).

$$sgn(t) = \begin{cases} 1 & ; \quad t > 0 \\ -1 & ; \quad t < 0 \end{cases}$$

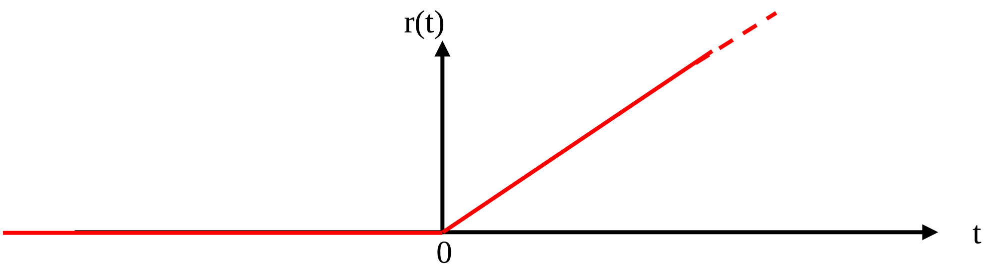


- It is a double step signal
- It is odd signal
- It is discontinuous at t = 0 $\Rightarrow \underset{t \to 0^-}{\text{Lt}} Sgn(t) \neq \underset{t \to 0^+}{\text{Lt}} Sgn(t)$
- Representation of u(t) by using sgn(t) is $u(t) = \frac{1+sgn(t)}{2}$
- Representation of sgn(t) by using u(t) is $sgn(t) = 2u(t) - 1$

3.4. Ramp Signal:

Unit ramp signal is denoted with r(t) and it can be defined as

$$r(t) = tu(t) = \begin{cases} t & ; \quad t \geq 0 \\ 0 & ; \quad t < 0 \end{cases}$$



- It is neither even nor odd signal
- Relation between u(t) and r(t) is $u(t) = \frac{d}{dt} r(t)$

4. Complex Exponential and Sinusoidal Signals:

- In general, the complex exponential signal is represented by

  $x(t) = Ae^{-jwt}, or\ y(t) = Ae^{jwt}$

  where, 'A' is Real or Imaginary or Complex number.

- Complex exponential signal can be represented in terms of Sinusoidal signal as

  $x(t) = Ae^{-jwt} = A[Cos(wt) - jSin(wt)]$

  $y(t) = Ae^{jwt} = A[Cos(wt) + jSin(wt)]$

  where, Sin(wt) and Cos(wt) are Sinusoidal signals

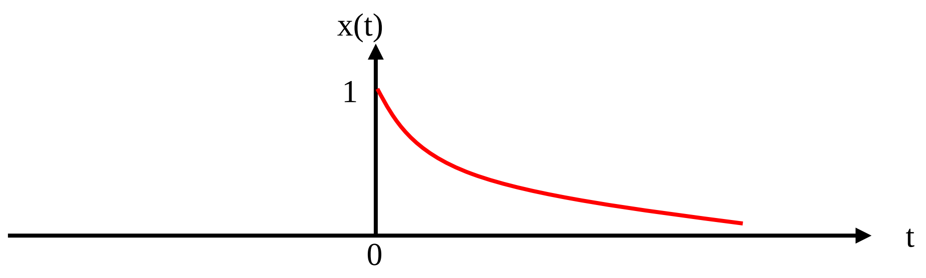
- Complex exponential and sinusoidal signals are periodic with a fundamental period,

  $T = \frac{2\pi}{w}$

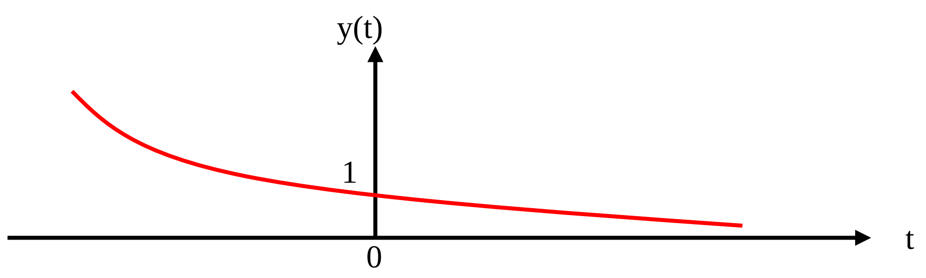
- $e^{-at}, e^{at}, e^{-at}u(t), e^{at}u(-t), and\ e^{-a|t|}$ are real exponential signals.

  where, 'a' is real constant.

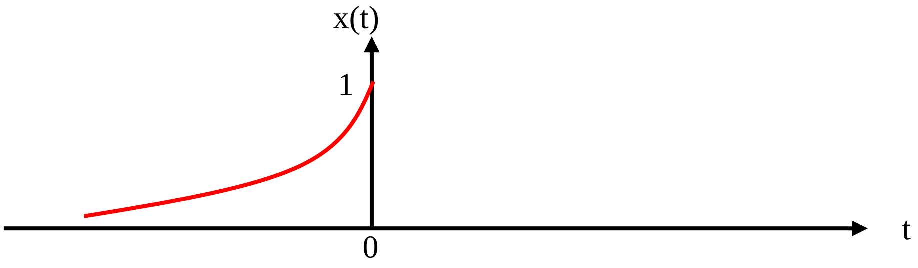
- Decaying exponential signal, $x(t) = e^{-at}u(t), a > 0$



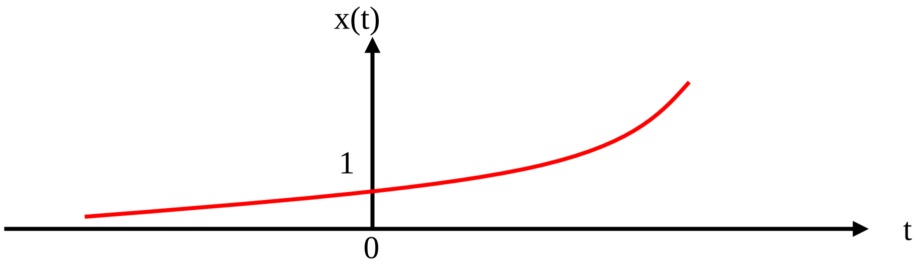
- Both sided decaying exponential, $y(t) = e^{-at}, a > 0$



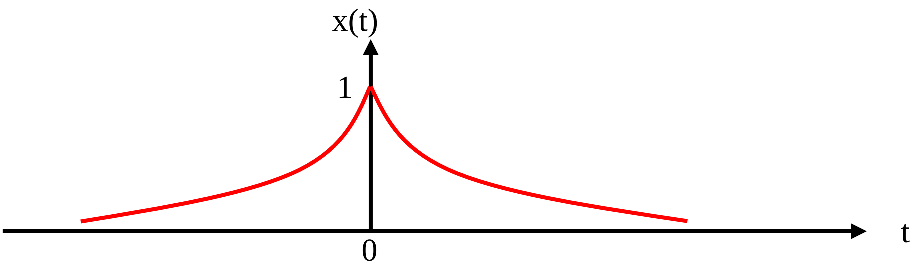
- Raising exponential signal, $x(t) = e^{at}u(-t), a > 0$



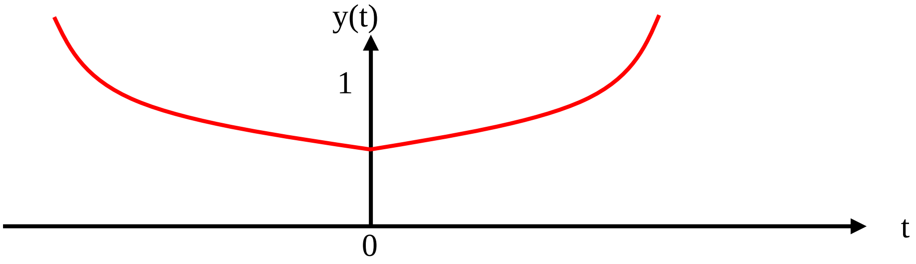
- Both sided raising exponential signal, $y(t) = e^{at}, a > 0$



- Double exponential signal, $x(t) = e^{-a|t|}, a > 0$



- Double exponential signal, $y(t) = e^{a|t|}, a > 0$



5. Operations on Signals:

Addition, Subtraction, Multiplication and Division are the few basic operations used in most of the occations. In addition to those basic operations, we have few more which are widely used in signal analysis;

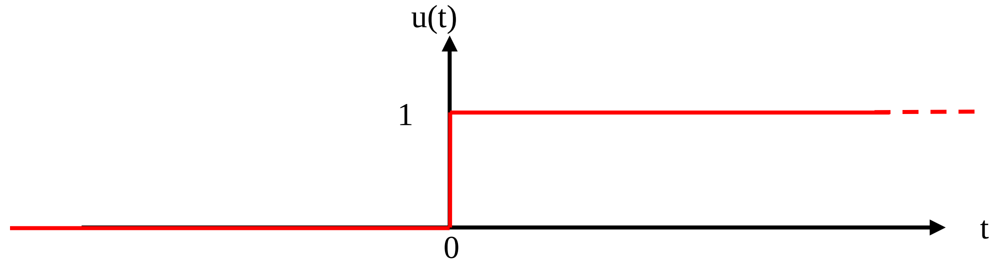
- Time Shifting
- Time Scaling
- Time Reversal or Folding
- Amplitude Shifting
- Amplitude Scaling

5.1. Time Shifting Operation:

If the time shifting operation is applied on a signal x(t), then the signal is shifted to left or right without changing its characteristics (width, amplitude and area). It is represented with $x(t \pm t_0)$, where t_0 is shift (delay or advance).

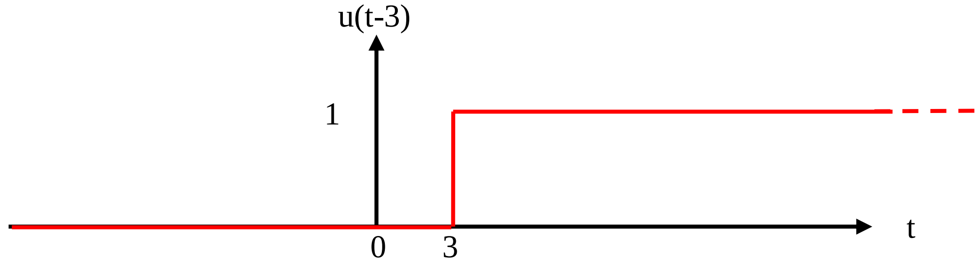
Example:

Given unit step signal, $x(t) = u(t) = \begin{cases} 1 & ; \ t > 0 \\ 0 & ; \ t < 0 \end{cases}$



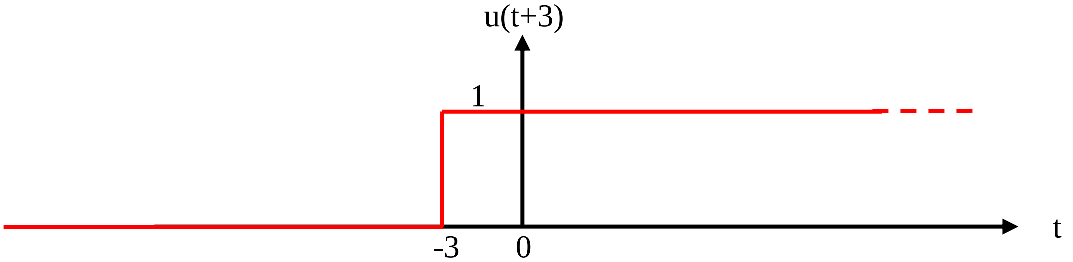
- Shifting form of x(t) by 3 units to right is represented with x(t-3)

$$x(t-3) = u(t-3) = \begin{cases} 1 & ; \ t > 3 \\ 0 & ; \ t < 3 \end{cases}$$



- Shifting form of x(t) by 3 units to left is represented with x(t+3)

$$x(t+3) = u(t+3) = \begin{cases} 1 & ; \ t > -3 \\ 0 & : \ t < -3 \end{cases}$$

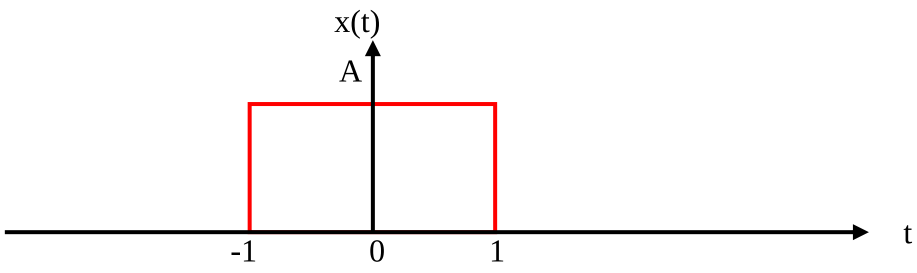


5.2. Time Scaling Operation:

If the time scaling operation is applied on a signal x(t), then the signal is compressed or expanded in time axes without changing its amplitude. It is represented with x(at), where 'a' is time scaling parameter. If a > 1, then it is compressed and if 0 < a < 1, then it is expanded signal.

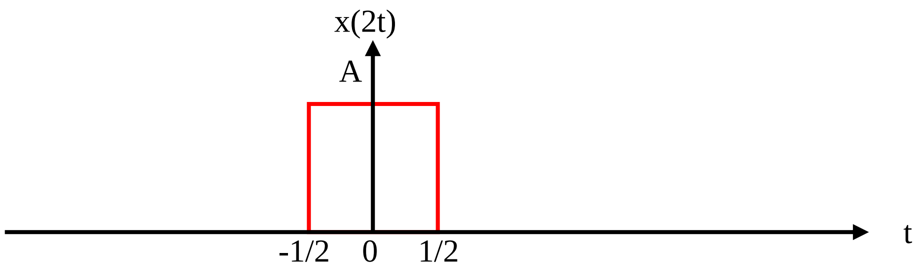
Example:

Given rectangular signal, $x(t) = A.rect\left(\frac{t}{2}\right) = \begin{cases} A, -1 < t < 1 \\ 0, Otherwise \end{cases}$



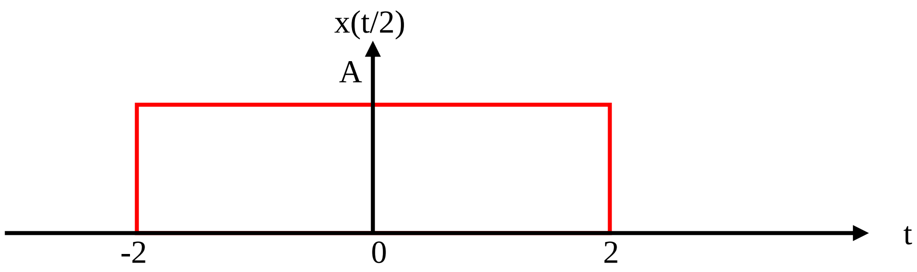
- Time Scaling (Compressed) form of x(t),

$$x(2t) = A.rect\left(\frac{2t}{2}\right) = A.rect\left(\frac{t}{1}\right) = \begin{cases} A, -\frac{1}{2} < t < \frac{1}{2} \\ 0, \ \ other\ wise \end{cases}, \text{ it is compressed signal}$$



- Time Scaling (Expanded) form of x(t),

$$x\left(\frac{t}{2}\right) = A.rect\left(\frac{t/2}{2}\right) = A.rect\left(\frac{t}{4}\right) = \begin{cases} A, -2 < t < 2 \\ 0, \ \ other\ wise \end{cases}, \text{ it is expanded signal}$$

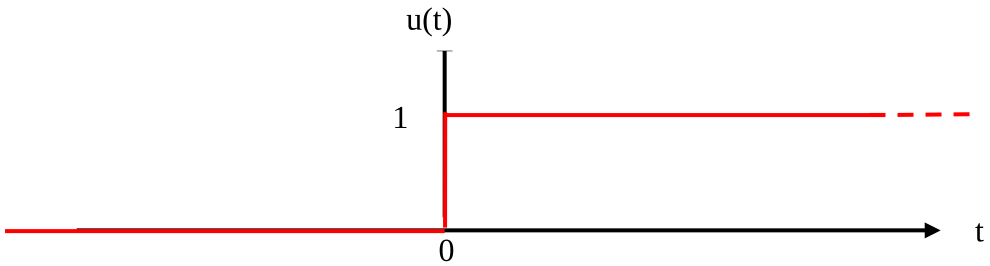


5.3. Time Reversal or Folding Operation:

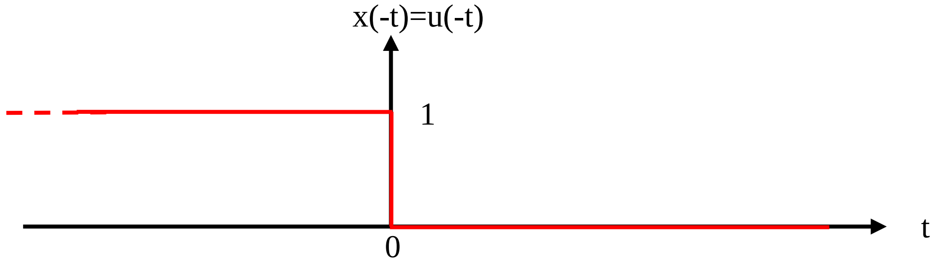
Time reversal signal can be obtained by interchanging left hand side and right hand side positions with respect to vertical axes or y-axes. If the given signal is x(t), then its time reversal form is represented with x(-t) and the operation is called folding or mirror image or time reversal.

Example-1:

Given unit step signal, $x(t) = u(t) = \begin{cases} 1 & ; \ t > 0 \\ 0 & ; \ t < 0 \end{cases}$

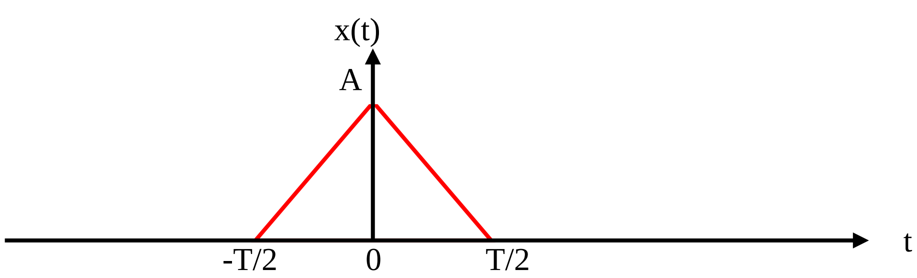


Time reversal or folding form of x(t) is $x(-t) = u(-t) = \begin{cases} 1 & ; \ t < 0 \\ 0 & ; \ t > 0 \end{cases}$



Example-2:

Given triangular signal, $x(t) = A.tri\left(\frac{t}{T}\right) = \begin{cases} A\left(1 - \frac{2}{T}|t|\right), -\frac{T}{2} < t < \frac{T}{2} \\ 0, \qquad other\ wise \end{cases}$



Time reversal or folding form of x(t) is $x(-t) = A.tri\left(\frac{-t}{T}\right) = \begin{cases} A\left(1 - \frac{2}{T}|t|\right), -\frac{T}{2} < t < \frac{T}{2} \\ 0, \qquad other\ wise \end{cases}$

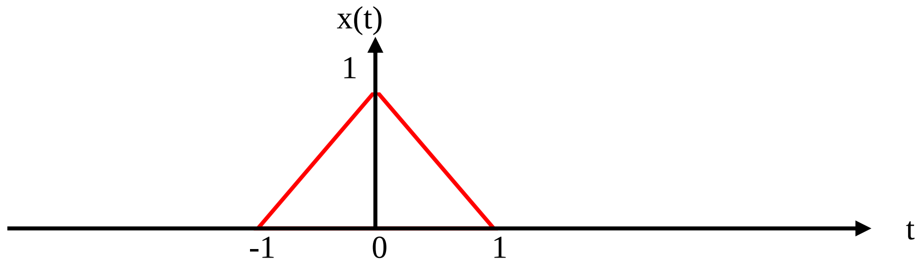
Given x(t) and x(-t) are same because it is even signal.

5.4. Amplitude Shifting Operation:

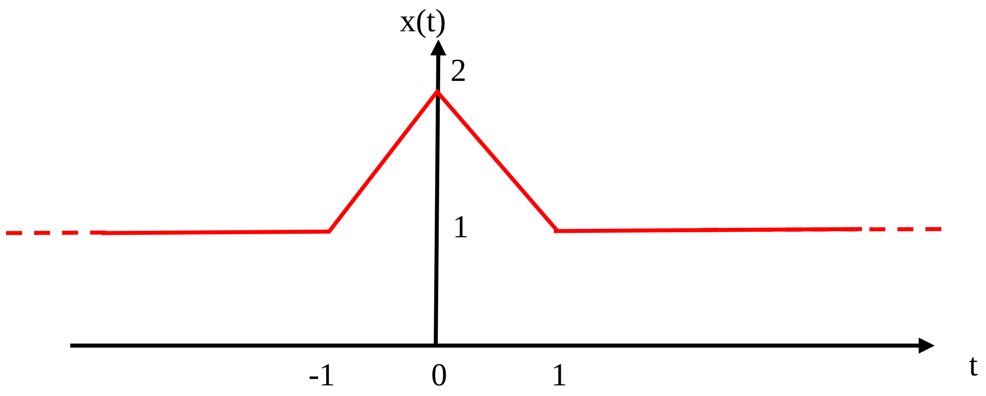
If the amplitude shifting operation is applied on a signal x(t), then the signal shifted to up or down. It is represented with 'A+x(t)', where 'A' is constant. It is also known as amplitude or DC level shifter

Example:

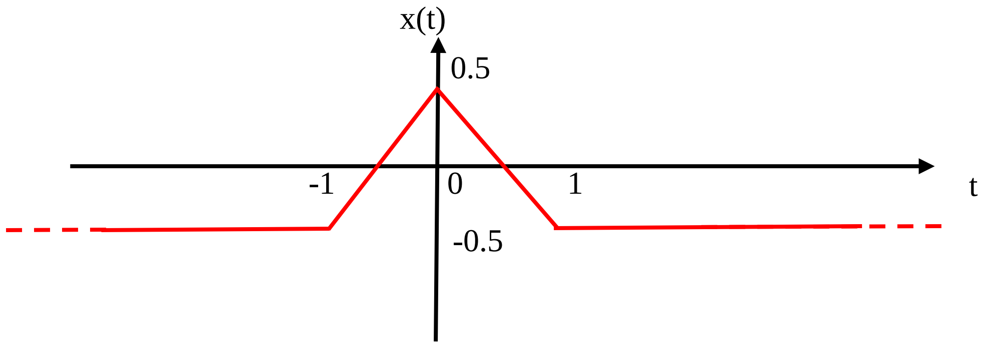
Given triangular signal, $x(t) = tri\left(\frac{t}{2}\right) = \begin{cases} 1 - |t|, -1 < t < 1 \\ 0, \quad\quad other\ wise \end{cases}$



- Amplitude shifting form of x(t), y(t) = 1 + x(t)



- Amplitude shifting form of x(t), y(t) = - 0.5 + x(t)

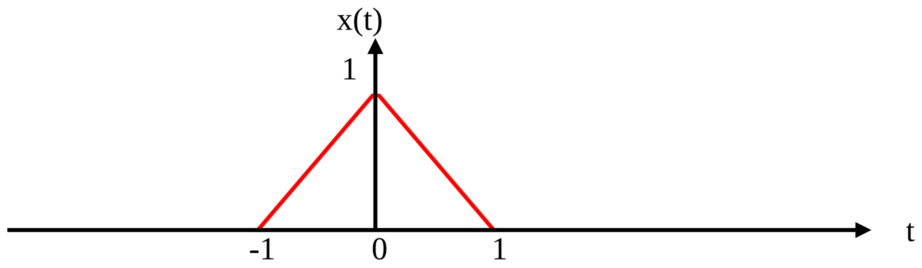


5.5. Amplitude Scaling Operation:

If the amplitude scaling operation is applied on a signal x(t), then the signal amplitude may increase or decrease without changing its duration. It is represented with Ax(t), where 'A' is amplitude scaling parameter.

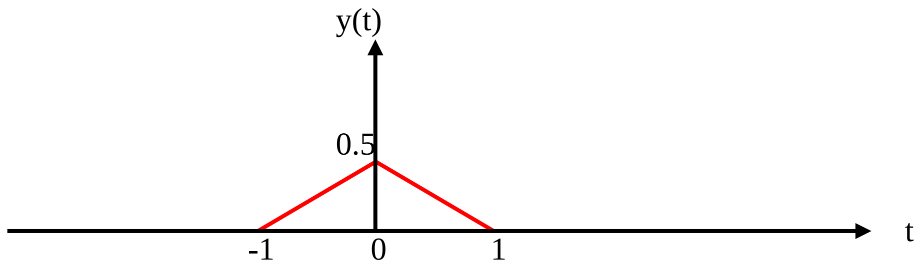
Example:

Given triangular signal, $x(t) = tri\left(\frac{t}{2}\right) = \begin{cases} 1-|t|, -1 < t < 1 \\ 0, \quad other\ wise \end{cases}$



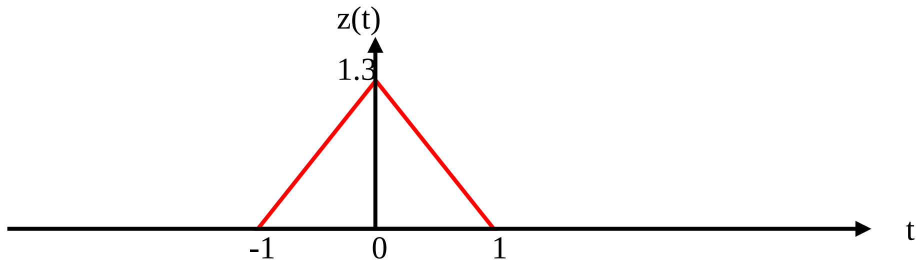
- Amplitude scaling form of given triangular signal is

$$y(t) = 0.5x(t) = 0.5tri\left(\frac{t}{2}\right) = \begin{cases} 0.5(1-|t|), -1 < t < 1 \\ 0, \quad other\ wise \end{cases}$$



- Amplitude scaling form of given triangular signal is

$$z(t) = 1.3x(t) = 1.3tri\left(\frac{t}{2}\right) = \begin{cases} 1.3(1-|t|), -1 < t < 1 \\ 0, \quad other\ wise \end{cases}$$



6. Properties or Characteristics or Classification of Signals:

Properties or Characteristics or Classification of Signals are

- Even and Odd Signals
- Causal and Noncausal Signals
- Bounded and Unbounded Signals
- Periodic and Aperiodic Signals
- Deterministic and Random Signals
- Energy and Power Signals

6.1. Even and Odd Signals:

- A signal x(t) is said to be even only when x(–t) = x(t). Even signals are symmetrical about y–axis, hence even signals are called symmetry signals.
- A signal x(t) is said to be odd only when x(–t) = – x(t). Odd signals are anti-symmetrical about y–axis, hence odd signals are called anti-symmetry signals.
- If the signal x(t) fails to satisfy even and odd property, then the signal is neither even nor odd and it can be expressed as the sum of even signal $x_e(t)$ and odd signal $x_o(t)$.

  $x(t) = x_e(t) + x_o(t)$ --------------- (1)

  Replace 't' with '–t'

  $x(-t) = x_e(-t) + x_o(-t) = x_e(t) - x_o(t)$ --------------- (2)

  (1) + (2) $\Rightarrow x(t) + x(-t) = x_e(t) + x_o(t) + x_e(t) - x_o(t) = 2\, x_e(t)$

  $$\Rightarrow x_e(t) = \frac{x(t) + x(-t)}{2}$$

  (1) - (2) $\Rightarrow x(t) - x(-t) = x_e(t) + x_o(t) - (x_e(t) - x_o(t)) = 2\, x_o(t)$

  $$\Rightarrow x_o(t) = \frac{x(t) - x(-t)}{2}$$

Note:

Condition for conjugate symmetry signal: $x^*(-t) = x(t)$.

Condition for conjugate anti-symmetry signal: $x^*(-t) = -x(t)$.

Conjugate symmetry and anti-symmetry signals can be computed from

$$x_{cs}(t) = \frac{x(t) + x^*(-t)}{2}$$

$$x_{cas}(t) = \frac{x(t) - x^*(-t)}{2}$$

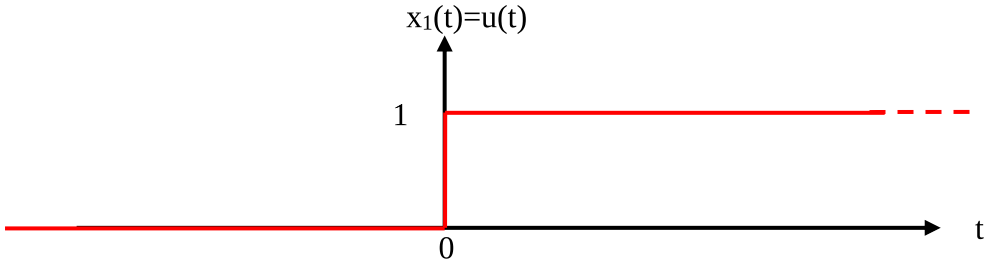
6.2. Causal and Noncausal Signals:

- A signal x(t) is said be **causal** only when x(t) = 0; for t < 0, that means causal signals are right sided.
- A signal x(t) is said be **anti-causal** only when x(t) = 0; for t > 0, that means anti-causal signals are left sided.
- A signal x(t) is said be **non-causal** only when x(t) ≠ 0; for t < 0, i.e. non-causal signals may be left sided or both sided and all anti-causal signals have come under non-causal signals.

Examples:

(a) $x_1(t) = u(t)$

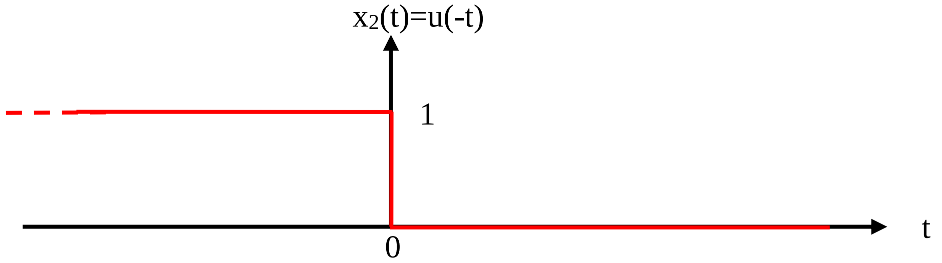
Given unit step signal, $x_1(t) = u(t) = 1, t > 0$



Given $x_1(t)$ is causal signal, because it is right sided (Extending from t=0 to ∞)

(b) $x_2(t) = u(-t)$

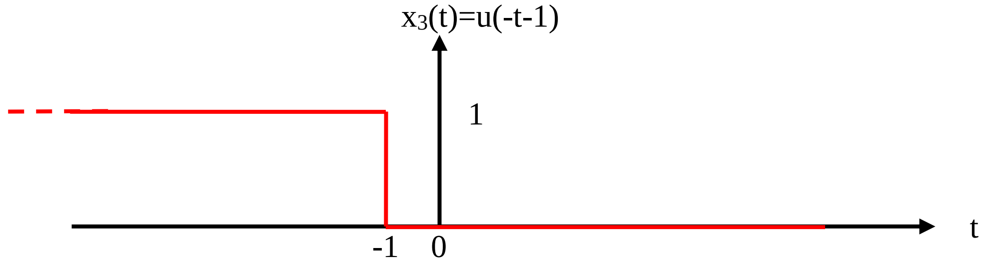
Given signal, $x_2(t) = u(-t) = 1, -t > 0 \; or \; t < 0$



Given $x_2(t)$ is anti-causal or non-causal signal, because it is left sided (Extending from t=-∞ to 0)

(c) $x_3(t) = u(-t-1)$

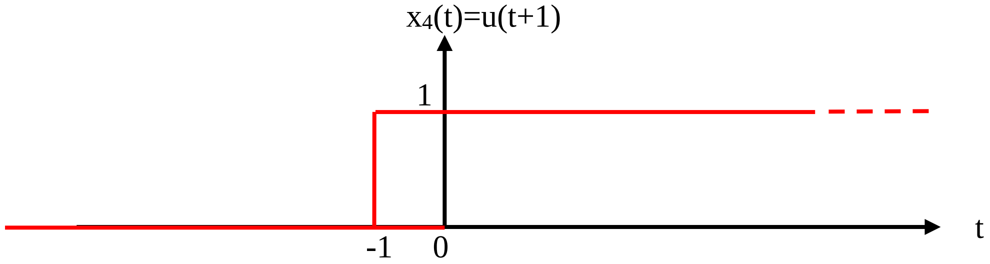
Given signal, $x_3(t) = u(-t-1) = 1, -t-1 > 0 \; or -t > 1 \; or \; t < -1$



Given $x_3(t)$ is anti-causal or non-causal signal, because it is left sided (Extending from t=-∞ to -1)

(d) $x_4(t) = u(t+1)$

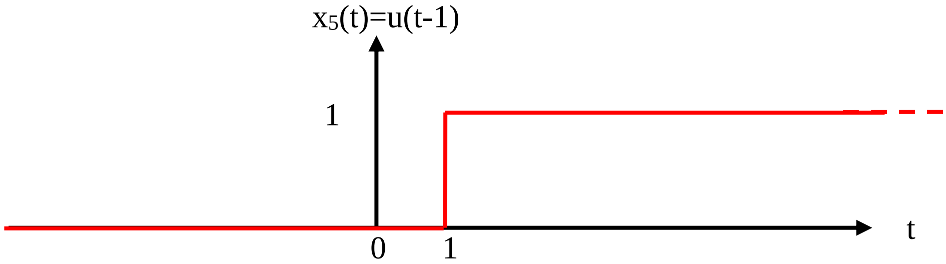
Given unit step signal, $x_4(t) = u(t+1) = 1, t+1 > 0 \; or \; t > -1$



Given $x_4(t)$ is non-causal signal, because it is both sided (Extending from t=-1 to ∞)

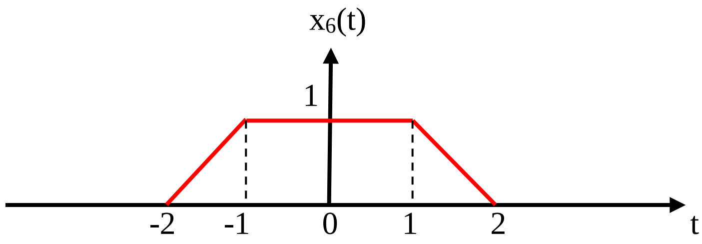
(e) $x_5(t) = u(t-1)$

Given unit step signal, $x_5(t) = u(t-1) = 1, t-1 > 0 \; or \; t > 1$



Given $x_5(t)$ is causal signal, because it is right sided (Extending from t=1 to ∞)

(f) $\boldsymbol{x_6(t) = r(t+2) - r(t+1) - [r(t-1) - r(t-2)]}$



Given $x_6(t)$ is non-causal signal, because it is both sided (Extending from t=-2 to 2)

$$(g) x_7(t) = \begin{cases} 0, & t < 0 \\ t, & 0 < t < 2 \\ 4 - t, & 2 < t < 4 \\ 0, & t > 4 \end{cases}$$

Given $x_7(t)$ is causal signal, because it is right sided (Extending from t=0 to 4)

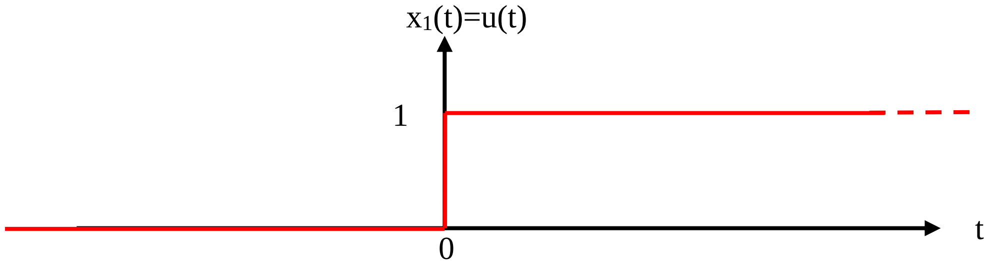
6.3. Bounded and Unbounded Signals:

A signal x(t) is said be bounded only when the amplitude of x(t) is finite for all values of 't' over the range $-\infty \le t \le \infty$. Condition for a bounded signal is $|x(t)| < \infty, -\infty \le t \le \infty$.

Examples:

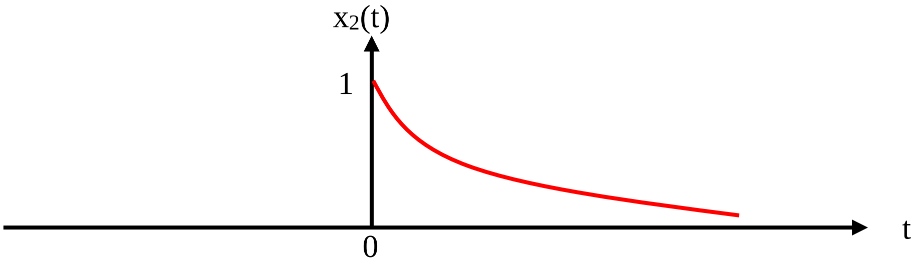
(a) $x_1(t) = u(t)$

Given unit step signal, $x_1(t) = u(t) = 1, t > 0$



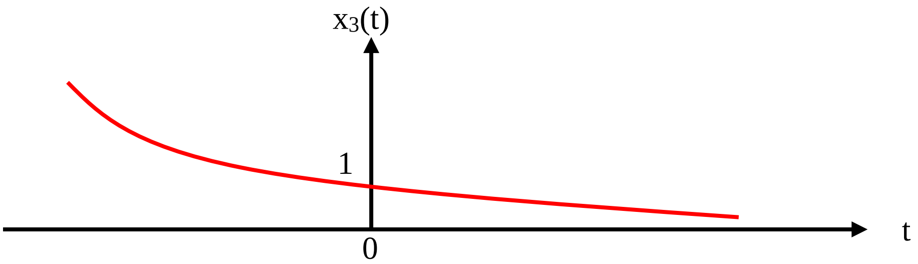
Given $x_1(t)$ is bounded signal, because its amplitude is finite for all values of t (limited to 0 and 1)

$(b) x_2(t) = e^{-2t}u(t)$



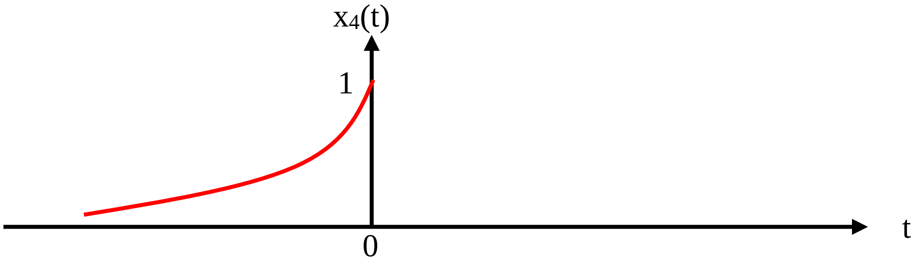
Given $x_2(t)$ is bounded signal, because its amplitude is finite for all values of t (limited to 0 and 1)

$(c) x_3(t) = e^{-2t}$



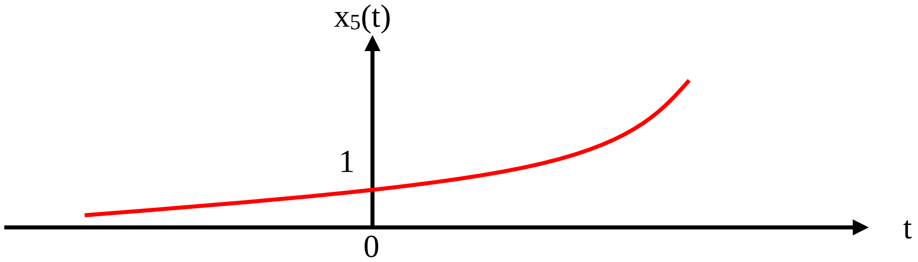
Given $x_3(t)$ is unbounded signal, because its amplitude is infinity at t= $-\infty$.

$(d) x_4(t) = e^{2t}u(-t)$



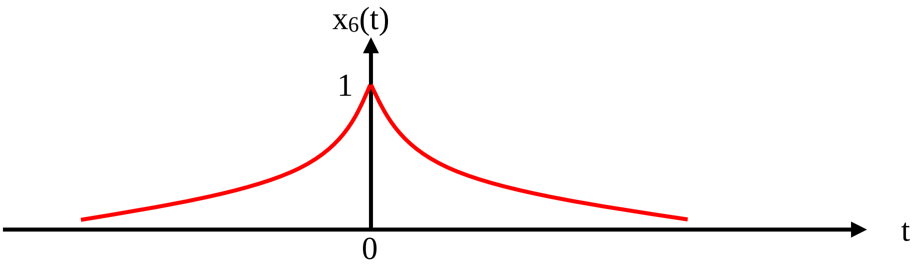
Given $x_4(t)$ is bounded signal, because its amplitude is finite for all values of t (limited to 0 and 1)

$(e) x_5(t) = e^{2t}$



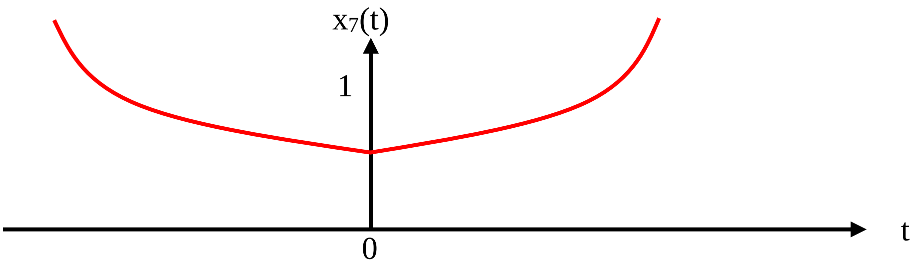
Given $x_5(t)$ is unbounded signal, because its amplitude is infinity at t= ∞.

$(f) x_6(t) = e^{-2|t|}$



Given $x_6(t)$ is bounded signal, because its amplitude is finite for all values of t (limited to 0 and 1)

$(g) x_7(t) = e^{2|t|}$



Given $x_7(t)$ is unbounded signal, because its amplitude is infinity at t= ±∞.

6.4. Periodic and Aperiodic Signals:

A signal x(t) is said be periodic if and only if x(t+T) = x(t), otherwise the signal is non-periodic or aperiodic, where T is called fundamental period or period of given signal x(t).

Note:

- Signals, $Sin(w_0t)$, $Cos(w_0t)$ and e^{jwot} are periodic with a period of $2\pi/w_0$.
- If $x_1(t)$ and $x_2(t)$ are periodic with periods T_1 and T_2, then $x_1(t)\pm x_2(t)$ is periodic only when T_1/T_2 is rational number.

6.5. Deterministic and Random Signals:

Signals that are completely specified by a mathematical expression are called deterministic signals, where the amplitude of the signal can be determined at any instant of time.

Examples:

- Unit Ramp Signal, $r(t) = tu(t) = \begin{cases} t & ; \ t > 0 \\ 0 & ; \ t < 0 \end{cases}$
- Decaying Exponential Signals, $x(t) = e^{-2t}u(t)$
- Raising Exponential Signals, $x(t) = e^{2t}u(-t)$

Signals whose characteristics are random in nature are called nondeterministic signals or random signals, where the mathematical representation is not possible, for example noise signal.

6.6. Energy and Power Signals:

- A signal x(t) is said to be energy signal only when the total energy (E) under the signal x(t) is finite and the average power (P) is zero.

  $0 < E < \infty$, and $P = 0$
- A signal x(t) is said to be power signal only when the average power (P) of the signal x(t) is finite and the total energy is infinity.

  $0 < P < \infty$, and $E = \infty$
- If a signal fails to satisfy energy and power signal properties, then the signal is called neither energy nor power.
- In general, most of the periodic signals are power signals and aperiodic signals are energy signals (Not all periodic signals are power signals and aperiodic signals are energy signals).
- Total Energy(E) and Average Power(P) of a signal x(t) can be computed from

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \ \& \ P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

7. Analogy between Vectors and Signals:

- Vector can be defined as a quantity, which has both magnitude and direction.
- Vector 'A' can be represented(approximated) in terms of another vector 'B' as; A = cB, where 'c' is constant and it can be computed by applying dot product;

  $A.B = cB.B \Rightarrow c = \frac{A.B}{B.B}$
- If A and B are perpendicular or orthogonal, then scalar or dot product is '0'.

  $i.e., A.B = 0 \Rightarrow c = \frac{A.B}{B.B} = 0$
- If A and B are perpendicular or orthogonal, then representation(approximation) of vector 'A' in terms vector 'B' is not possible.
- If A and B are parallel, then very good approximation is possible with zero error.
- In XY-plane(2D), vector 'A' can be represented as; $A = xa_x + ya_y$, where a_x and a_y are unit vectors along X and Y directions.
- In XYZ-plane(3D), vector 'A' can be represented as; $A = xa_x + ya_y + za_z$, where a_x, a_y and a_z are unit vectors along X, Y and Z directions. It is called orthogonal vector space.
- i.e., In orthogonal vector space, a vector 'A' can be represented as the sum of mutually orthogonal vectors.
- We can extend all the above vector concepts to signals also, because all signals are analogous to vectors.
- A real valued signal x(t) over the interval (t_1, t_2) can be approximated by another signal g(t) over the same interval (t_1, t_2) as; x(t) = C g(t), where 'C' is the approximation constant.
- Signals x(t) and g(t) are orthogonal over the interval (t_1, t_2), then $\int_{t_1}^{t_2} x(t)g(t)dt = 0$.

8. Approximation of a signal by another Signal:

A real valued signal x(t) over the interval (t_1, t_2) can be approximated by another signal g(t) over the same interval (t_1, t_2) is

$$x(t) = C\, g(t)$$

Where, C is the approximation constant.

The difference between given x(t) and the approximated x(t) is called error signal. It is represented with $x_e(t)$.

$$x_e(t) = x(t) - C\, g(t)$$

8.1. Mean Square Error (MSE):

In every approximation process, there will be some error and error due to the approximation can be computed through Mean Square Error (MSE). It is denoted with ε and it can be computed from the formula

$$\epsilon = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}[x_e(t)]^2 dt$$

Substitute, $x_e(t) = x(t) - C\,g(t)$ and develop an expression to compute the mean square error

$$\epsilon = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}[x(t) - Cg(t)]^2 dt$$

$$\epsilon = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}[x^2(t) + C^2 g^2(t) - 2Cx(t)g(t)] dt$$

$$\epsilon = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt + C^2\int_{t_1}^{t_2} g^2(t)dt - 2C\int_{t_1}^{t_2} x(t)g(t)dt\right) - - - - (1)$$

For good approximation, the mean square error (ε) should be minimum. Compute the constant (C) by differentiating ε with respect to 'C' and equate to zero

$$\frac{d\epsilon}{dC} = 0$$

$$\Rightarrow \frac{1}{t_2 - t_1}\left(0 + 2C\int_{t_1}^{t_2} g^2(t)dt - 2\int_{t_1}^{t_2} x(t)g(t)dt\right) = 0$$

$$\Rightarrow 2C\int_{t_1}^{t_2} g^2(t)dt = 2\int_{t_1}^{t_2} x(t)g(t)dt$$

$$\Rightarrow C = \frac{\int_{t_1}^{t_2} x(t)g(t)dt}{\int_{t_1}^{t_2} g^2(t)dt} = \frac{\int_{t_1}^{t_2} x(t)g(t)dt}{K} - - - - - - - - - - - - - - - (2)$$

Where, $K = \int_{t_1}^{t_2} g^2(t)dt$ and $\int_{t_1}^{t_2} x(t)g(t)dt = CK - - - - - - - - - - - (3)$

Evaluate the mean square error (ε) by substituting equation (3) in equation (1)

$$\epsilon = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt + C^2(K) - 2C(CK)\right)$$

$$\epsilon = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt + C^2K - 2C^2K\right)$$

$$\epsilon = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt - C^2K\right)$$

8.2. Condition for Orthogonal Signals:

- If A and B are orthogonal, then scalar or dot product is '0'.

  A•B = 0

- We know that signals are analogous to vectors, i.e if two signals x(t) and g(t) are orthogonal over the interval (t_1, t_2), then

  $$\int_{t_1}^{t_2} x(t)g(t)dt = 0$$

  It is the condition for orthogonal signals.

- All sinusoidal signals, sin(wt) and cos(wt) are orthogonal signals.
- All complex exponential signals, $x(t) = Ae^{-jwt}, and\ y(t) = Ae^{jwt}$ are orthogonal signals.

9. Orthogonal Signal Space:

- In XYZ-plane(3D), vector 'A' can be represented as; A = $xa_x + ya_y + za_z$, where a_x, a_y and a_z are unit vectors along X, Y and Z directions. It is called orthogonal vector space.
- i.e., In orthogonal vector space, a vector 'A' can be represented as the sum of mutually orthogonal vectors.
- We know that signals are analogous to vectors, i.e A real valued signal x(t) over the interval (t_1, t_2) can be approximated by a set of mutually orthogonal signals $g_1(t)$, $g_2(t)$, ……… $g_n(t)$ over the same interval (t_1, t_2) is

  $x(t) = C_1\, g_1(t) + C_2\, g_2(t) + \ldots\ldots\ldots\ C_r\, g_r(t) + \ldots\ldots\ldots\ldots\ldots\ C_n\, g_n(t)$

  $$x(t) = \sum_{r=1}^{n} C_r g_r(t)$$

  where, C_1, C_2, ………. C_r, …………. C_n are approximation constants.

- The above concept comes under orthogonal signal space.

10. Approximation of a signal by a set of mutually orthogonal Signals:

A real valued signal x(t) over the interval (t_1, t_2) can be approximated by a set of mutually orthogonal signals $g_1(t)$, $g_2(t)$, ……… $g_n(t)$ over the same interval (t_1, t_2) is

$x(t) = C_1\, g_1(t) + C_2\, g_2(t) + \ldots\ldots\ldots\ C_r\, g_r(t) + \ldots\ldots\ldots\ldots\ldots\ C_n\, g_n(t)$

$$x(t) = \sum_{r=1}^{n} C_r g_r(t)$$

where, C_1, C_2, ………. C_r, …………. C_n are approximation constants. the difference between given x(t) and the approximated x(t) is called error signal. It is represented with $x_e(t)$.

$$x_e(t) = x(t) - \sum_{r=1}^{n} C_r g_r(t)$$

10.1. Mean Square Error (MSE):

It is denoted with ε_r and it can be computed from the formula

$$\epsilon_r = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}[x_e(t)]^2 dt$$

$$= \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}\left(x(t) - \sum_{r=1}^{n} C_r g_r(t)\right)^2 dt$$

$$= \frac{1}{t_2-t_1}\int_{t_1}^{t_2}(x^2(t) + (\sum_{r=1}^{n} C_r g_r(t))^2 - 2x(t)\sum_{r=1}^{n} C_r g_r(t))dt$$

$$= \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt + \int_{t_1}^{t_2}\left(\sum_{r=1}^{n} C_r g_r(t)\right)^2 dt - \int_{t_1}^{t_2} 2x(t)\sum_{r=1}^{n} C_r g_r(t)\, dt\right)$$

$$= \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt + \sum_{r=1}^{n}\left({C_r}^2\int_{t_1}^{t_2}{g_r}^2(t)dt\right) - 2\sum_{r=1}^{n} C_r\int_{t_1}^{t_2} x(t)g_r(t)dt\right) - - - (1)$$

For good approximation, the mean square error (ε_r) should be minimum. Compute the constant (Cr) by differentiating ε_r with respect to 'Cr' and equate to zero

$$\Rightarrow \frac{d\epsilon_r}{dC_r} = 0$$

$$\Rightarrow \frac{1}{t_2 - t_1}\left(0 + 2C_r\int_{t_1}^{t_2}{g_r}^2(t)dt - 2\int_{t_1}^{t_2} x(t)g_r(t)dt\right) = 0$$

$$\Rightarrow 2C_r\int_{t_1}^{t_2}{g_r}^2(t)dt = 2\int_{t_1}^{t_2} x(t)g_r(t)dt$$

$$\Rightarrow C_r = \frac{\int_{t_1}^{t_2} x(t)g_r(t)dt}{\int_{t_1}^{t_2}{g_r}^2(t)dt} = \frac{\int_{t_1}^{t_2} x(t)g_r(t)dt}{k_r} - - - - - - - - - - - - - - - (2)$$

Where, $k_r = \int_{t_1}^{t_2}{g_r}^2(t)dt$ and $\int_{t_1}^{t_2} x(t)g_r(t)dt = C_r k_r - - - - - - - - - - (3)$

Evaluate the mean square error (ε_r) by substituting equation (3) in equation (1)

$$\epsilon_r = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt + \sum_{r=1}^{n}{C_r}^2(K_r) - 2\sum_{r=1}^{n} C_r(C_r K_r)\right)$$

$$\epsilon_r = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt + \sum_{r=1}^{n}{C_r}^2 K_r - 2\sum_{r=1}^{n}{C_r}^2 K_r\right)$$

$$\epsilon_r = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt - \sum_{r=1}^{n}{C_r}^2 K_r\right)$$

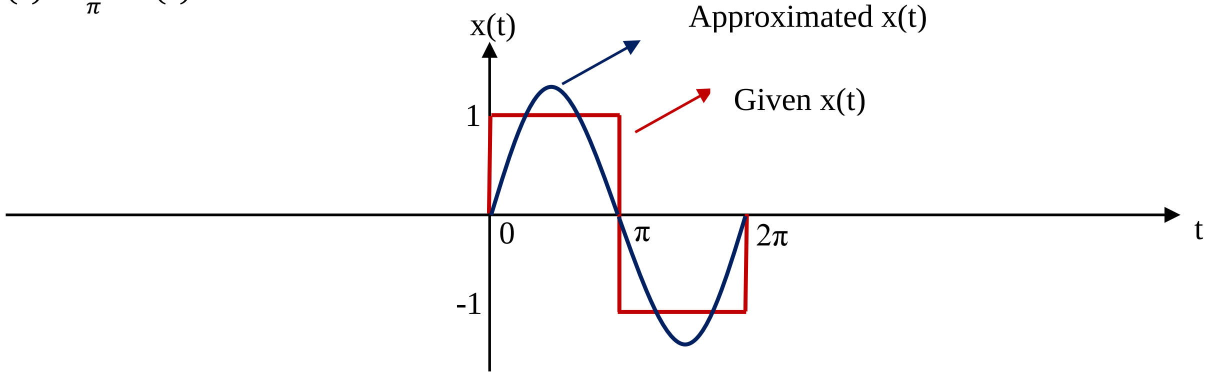
10.2. Gibb's Phenomena:

Let us consider the following approximation process in different cases

Case-1:

If the rectangular signal $x(t) = \begin{cases} 1, & 0 < t < \pi \\ -1, & \pi < t < 2\pi \end{cases}$ is approximated with sinusoidal signals g(t)=Sin(t) or g(t)=Sin(t), Sin(2t), then the approximated signal for minimum error is

$$x(t) = \frac{4}{\pi} Sin(t)$$

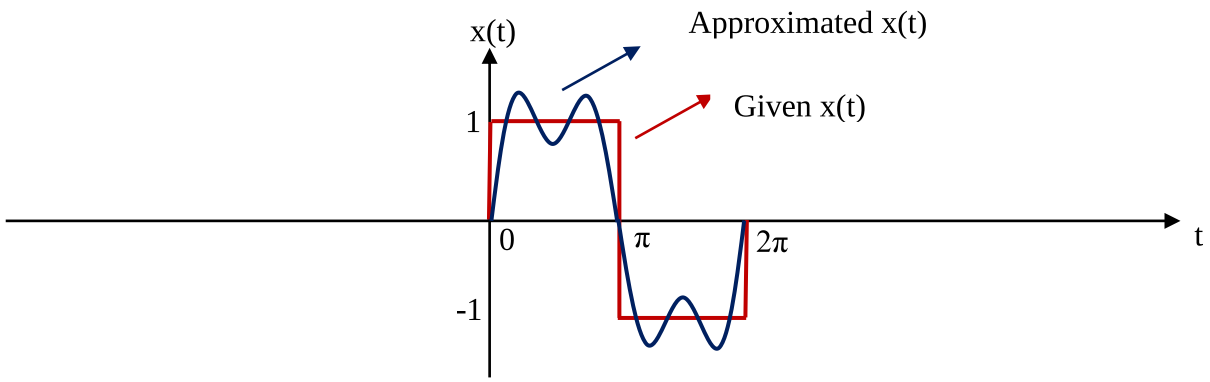


Error in the approximation process, $\epsilon = 1 - \frac{8}{\pi^2} = 0.189 \ \ or\ 18.9\%$

Case-2:

If the rectangular signal $x(t) = \begin{cases} 1, & 0 < t < \pi \\ -1, & \pi < t < 2\pi \end{cases}$ is approximated with sinusoidal signals g(t)=Sin(t), Sin(2t), Sin(3t) or g(t)=Sin(t), Sin(2t), Sin(3t), Sin(4t), then the approximated signal for minimum error is

$$x(t) = \frac{4}{\pi} Sin(t) + \frac{4}{3\pi} Sin(3t)$$

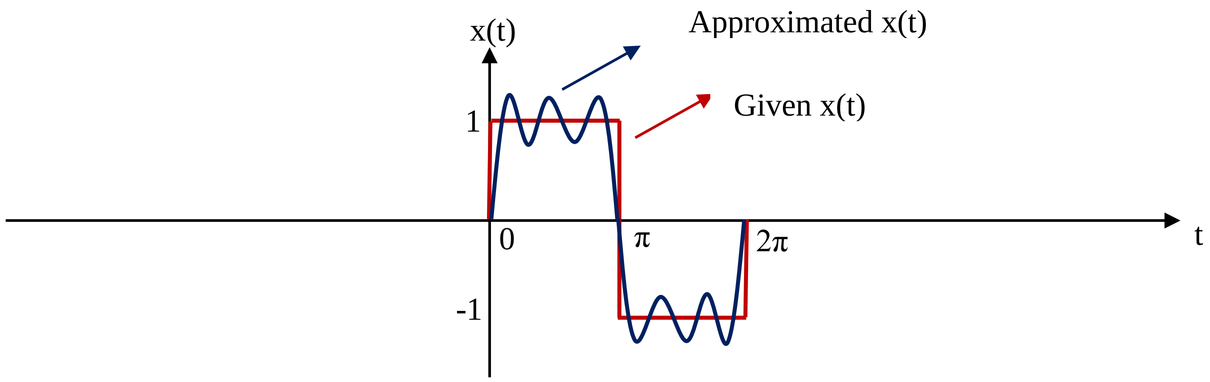


Error in the approximation process, $\epsilon = 1 - \frac{8}{\pi^2} - \frac{8}{9\pi^2} = 0.0989 \ \ or\ 9.89\%$

Case-3:

If the rectangular signal $x(t) = \begin{cases} 1, & 0 < t < \pi \\ -1, & \pi < t < 2\pi \end{cases}$ is approximated with sinusoidal signals g(t)=Sin(t), Sin(2t), Sin(3t), Sin(4t), Sin(5t) or g(t)=Sin(t), Sin(2t), Sin(3t), Sin(4t), Sin(5t), Sin(6t), then the approximated signal for minimum error is

$$x(t) = \frac{4}{\pi}Sin(t) + \frac{4}{3\pi}Sin(3t) + \frac{4}{5\pi}Sin(5t)$$

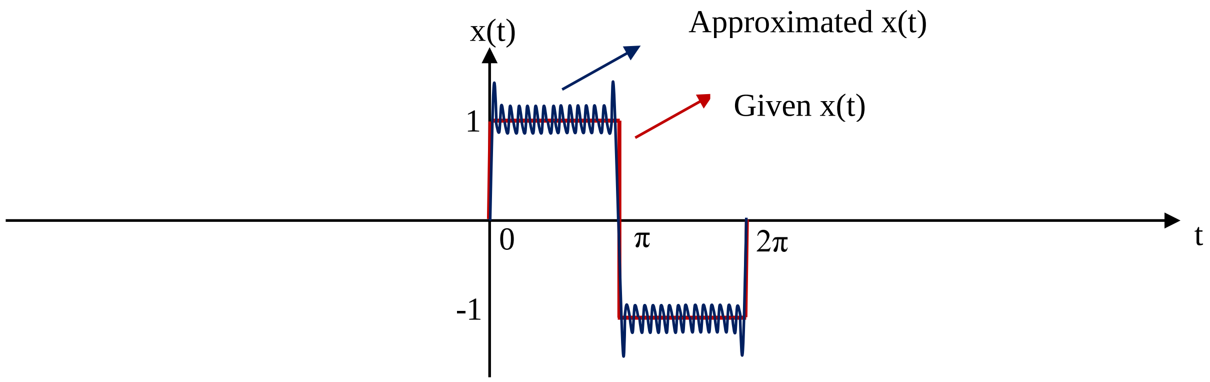


Error in the approximation process, $\epsilon = 1 - \frac{8}{\pi^2} - \frac{8}{9\pi^2} - \frac{8}{25\pi^2} = 0.0665 \;\; or\; 6.65\%$

Case-N:

If the rectangular signal $x(t) = \begin{cases} 1, & 0 < t < \pi \\ -1, & \pi < t < 2\pi \end{cases}$ is approximated with more number of sinusoidal signals g(t)=Sin(t), Sin(2t), Sin(3t), Sin(4t), Sin(5t), Sin(6t), Sin(7t), Sin(8t),…. then the approximated signal for minimum error is

$$x(t) = \frac{4}{\pi}Sin(t) + \frac{4}{3\pi}Sin(3t) + \frac{4}{5\pi}Sin(5t) + \frac{4}{7\pi}Sin(7t) + \cdots$$



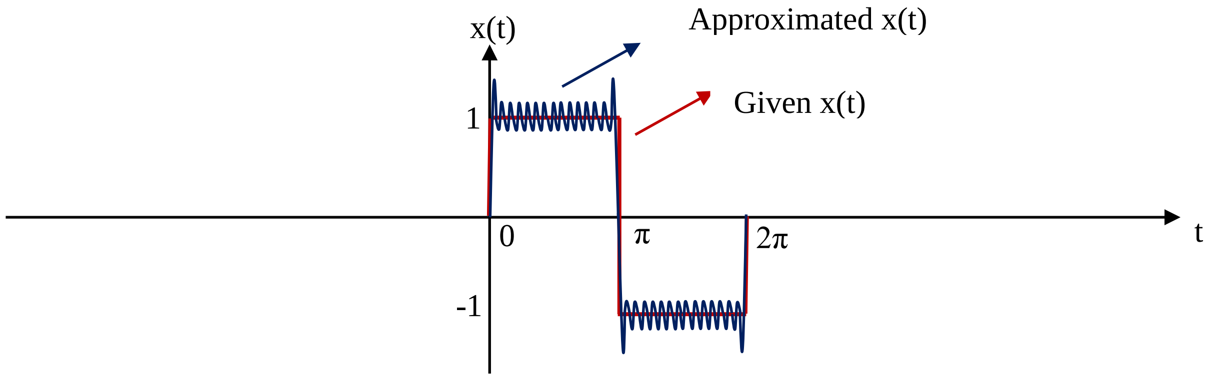
Error in the approximation process, $\epsilon = 1 - \frac{8}{\pi^2} - \frac{8}{9\pi^2} - \frac{8}{25\pi^2} - \frac{8}{49\pi^2} - \cdots = 0$

Conclusion – Gibb's Phenomena:

It is very clear from above approximation process (Case-1, Case-2, Case-3,….,Case-N). If the rectangular signal x(t) is approximated with a sinusoidal signal g(t), then the error in the approximation process is more. To reduce the error, we can go for the approximation of x(t) by using more number of sinusoidal signals. i.e if the rectangular signal x(t) is approximated with more number of sinusoidal orthogonal signals, then the error reduced to minimum (zero) and the pass band and stop band of approximated signal contains oscillations and approximated x(t) may have discontinuities at t=0, π, 2π, ….. Those oscillations are called Gibb's oscillations and the effect of getting oscillations and discontinuities due to the approximation of given rectangular signal x(t) with more number of sinusoidal orthogonal signals is called Gibb's phenomena.

If the rectangular signal $x(t) = \begin{cases} 1, & 0 < t < \pi \\ -1, & \pi < t < 2\pi \end{cases}$ is approximated with more number of sinusoidal signals g(t)=Sin(t), Sin(2t), Sin(3t), Sin(4t), Sin(5t), Sin(6t), Sin(7t), Sin(8t),…. then the approximated signal for minimum error is

$$x(t) = \frac{4}{\pi}Sin(t) + \frac{4}{3\pi}Sin(3t) + \frac{4}{5\pi}Sin(5t) + \frac{4}{7\pi}Sin(7t) + \cdots$$



Error in the approximation process, $\epsilon = 1 - \frac{8}{\pi^2} - \frac{8}{9\pi^2} - \frac{8}{25\pi^2} - \frac{8}{49\pi^2} - \cdots = 0$

11. Orthogonality in Complex Signals:

We know that a real valued signal x(t) can be approximated by using another real valued signal g(t) or a set of orthogonal real valued signals. Now we can extend all those concepts to complex signals.

11.1. Approximation of a complex signal by another complex signal:

A complex signal x(t) over the interval (t_1, t_2) can be approximated by another complex signal g(t) over the same interval (t_1, t_2) is

$$x(t) = C\, g(t)$$

where, C is the approximation constant.

The difference between given x(t) and the approximated x(t) is called error signal. It is represented with $x_e(t)$.

$$x_e(t) = x(t) - C\, g(t)$$

In every approximation process, there will be some error and error due to the approximation can be computed through Mean Square Error (MSE). It is denoted with ε and it can be computed from the formula

$$\epsilon = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} [x_e(t)]^2 dt$$

For good approximation, the mean square error (ε) should be minimum, for the minimum error, the constant (C) of approximation is

$$C = \frac{\int_{t_1}^{t_2} x(t)g^*(t)dt}{\int_{t_1}^{t_2} g(t)g^*(t)dt}$$

Note: If complex signals x(t) and g(t) are orthogonal, then $\int_{t_1}^{t_2} x(t)g^*(t)dt = 0$

11.2. Approximation of a complex signal by set of mutually orthogonal complex signals

A complex signal x(t) over the interval (t_1, t_2) can be approximated by a set of mutually orthogonal complex signals $g_1(t)$, $g_2(t)$, ……… $g_n(t)$ over the same interval (t_1, t_2) is

$x(t) = C_1\, g_1(t) + C_2\, g_2(t) + \ldots\ldots\ldots\ C_r\, g_r(t) + \ldots\ldots\ldots\ldots\ldots\ C_n\, g_n(t)$

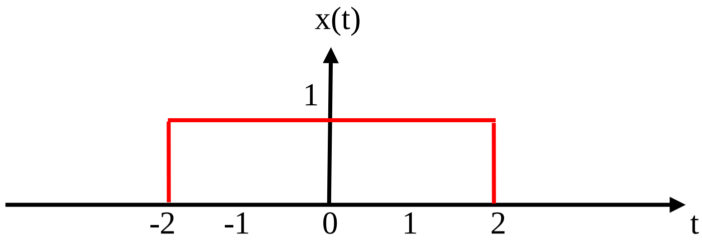
$$x(t) = \sum_{r=1}^{n} C_r g_r(t)$$

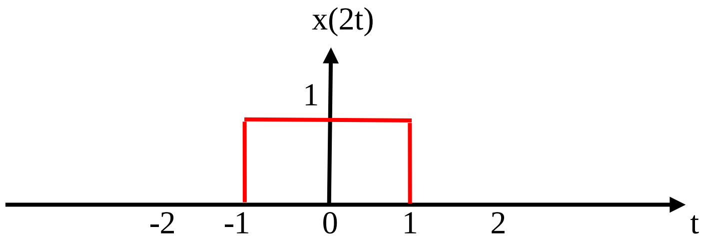
For good approximation, the constant of approximation is

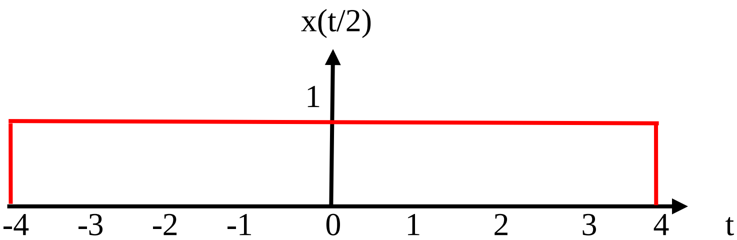
$$C_r = \frac{\int_{t_1}^{t_2} x(t)g_r^*(t)dt}{\int_{t_1}^{t_2} g_r(t)g_r^*(t)dt}$$

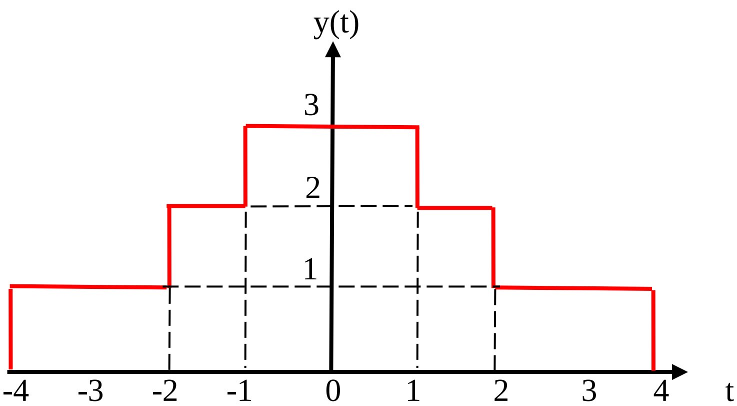
12. Solved Problems:

(12.1) Draw the graphical form of y(t) = x(t) + x(2t) + x(t/2) and represent by using unit step signal u(t).







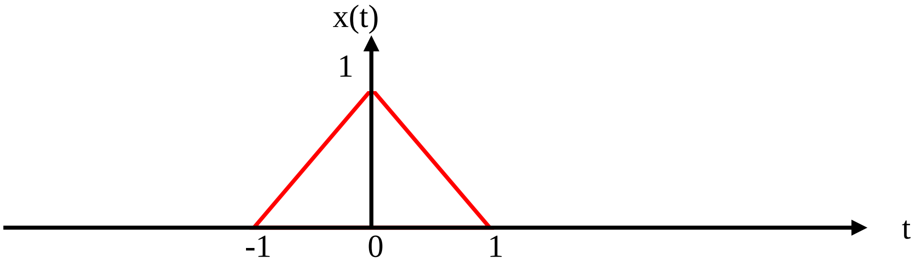


y(t) = u(t+4)-u(t+2)+2[u(t+2)-u(t+1)]+3[u(t+1)-u(t-1)]+2[u(t-1)-u(t-2)]+u(t-2)-u(t-4)

= u(t+4)-u(t+2)+2u(t+2)-2u(t+1)]+3u(t+1)-3u(t-1)+2u(t-1)-2u(t-2)+u(t-2)-u(t-4)

= u(t+4)+u(t+2)+u(t+1)-u(t-1)-u(t-2)-u(t-4)

(12.2) Draw the graphical form of a signal y(t) = 2x(3t-4).



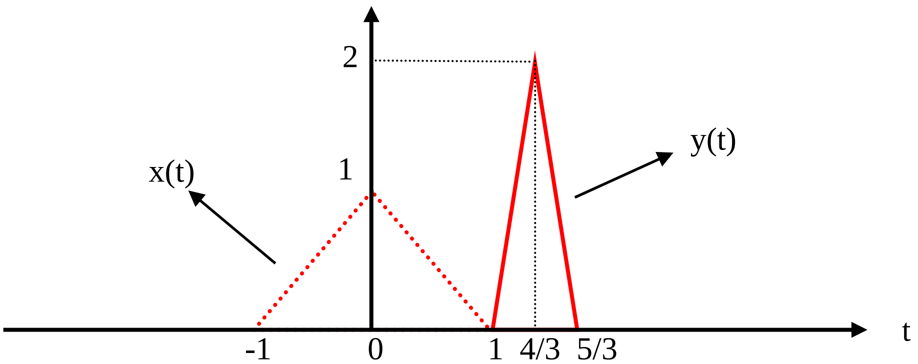
Given x(t) is extending from -1 to 1 and it is symmetrical about t=0.

$y(t) = 2x(3t-4) = 2x\left(3\left(t-\frac{4}{3}\right)\right)$ signal includes three operations

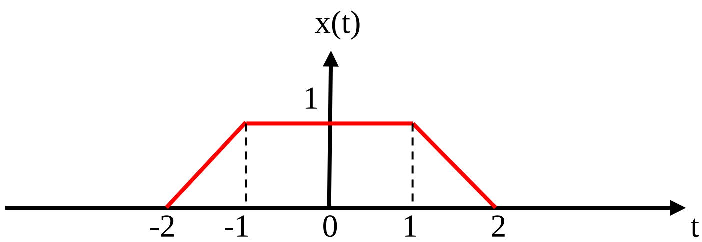
- Amplitude scaling (Amplitude doubles)
- Time scaling (Compressed in time axes by 3)
- Time shifting (Shifting in time axes by 4/3)

Procedure to find locations or positions

- Starting location $\Rightarrow 3t-4=-1 \Rightarrow 3t=3 \Rightarrow t=\frac{3}{3}=1$
- Centre location $\Rightarrow 3t-4=0 \Rightarrow 3t=4 \Rightarrow t=\frac{4}{3}=1.33$
- Ending location $\Rightarrow 3t-4=1 \Rightarrow 3t=5 \Rightarrow t=\frac{5}{3}=1.66$

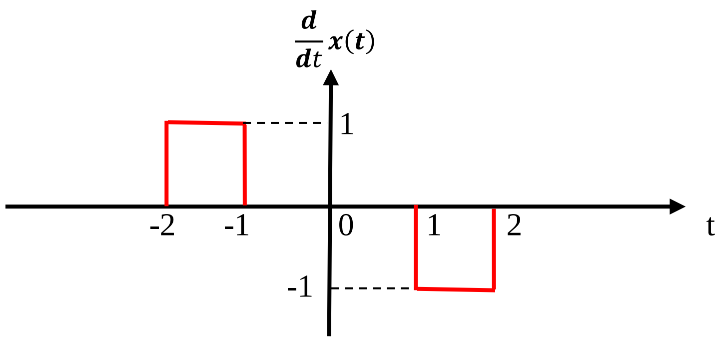


(12.3) Evaluate the integral $\int_{-\infty}^{\infty}\left|\frac{d^2}{dt^2}x(t)\right| dt$

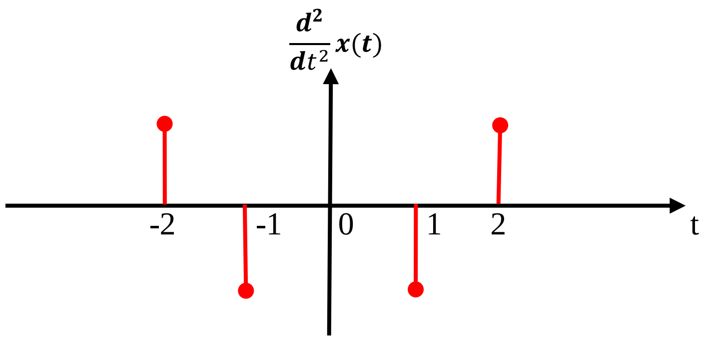


$$\begin{aligned} x(t) &= (t+2)[u(t+2)-u(t+1)] + u(t+1) - u(t-1) + (-t+2)[u(t-1)-u(t-2)] \\ &= (t+2)u(t+2) - (t+2-1)u(t+1) - (1+t-2)(u(t-1) - (-t+2)u(t-2) \\ &= (t+2)u(t+2) - (t+1)u(t+1) - (t-1)(u(t-1) + (t-2)u(t-2) \\ &= r(t+2) - r(t+1) - [r(t-1) - r(t-2)] \end{aligned}$$

$$\begin{aligned} \frac{d}{dt}x(t) &= \frac{d}{dt}r(t+2) - \frac{d}{dt}r(t+1) - \left(\frac{d}{dt}r(t-1) - \frac{d}{dt}r(t-2)\right) \\ &= u(t+2) - u(t+1) - \big(u(t-1) - u(t-2)\big) \end{aligned}$$



$$\begin{aligned} \frac{d^2}{dt^2}x(t) &= \frac{d}{dt}\left(\frac{d}{dt}x(t)\right) = \frac{d}{dt}u(t+2) - \frac{d}{dt}u(t+1) - \left(\frac{d}{dt}u(t-1) - \frac{d}{dt}u(t-2)\right) \\ &= \delta(t+2) - \delta(t+1) - (\delta(t-1) - \delta(t-2)) \\ &= \delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t-2) \end{aligned}$$

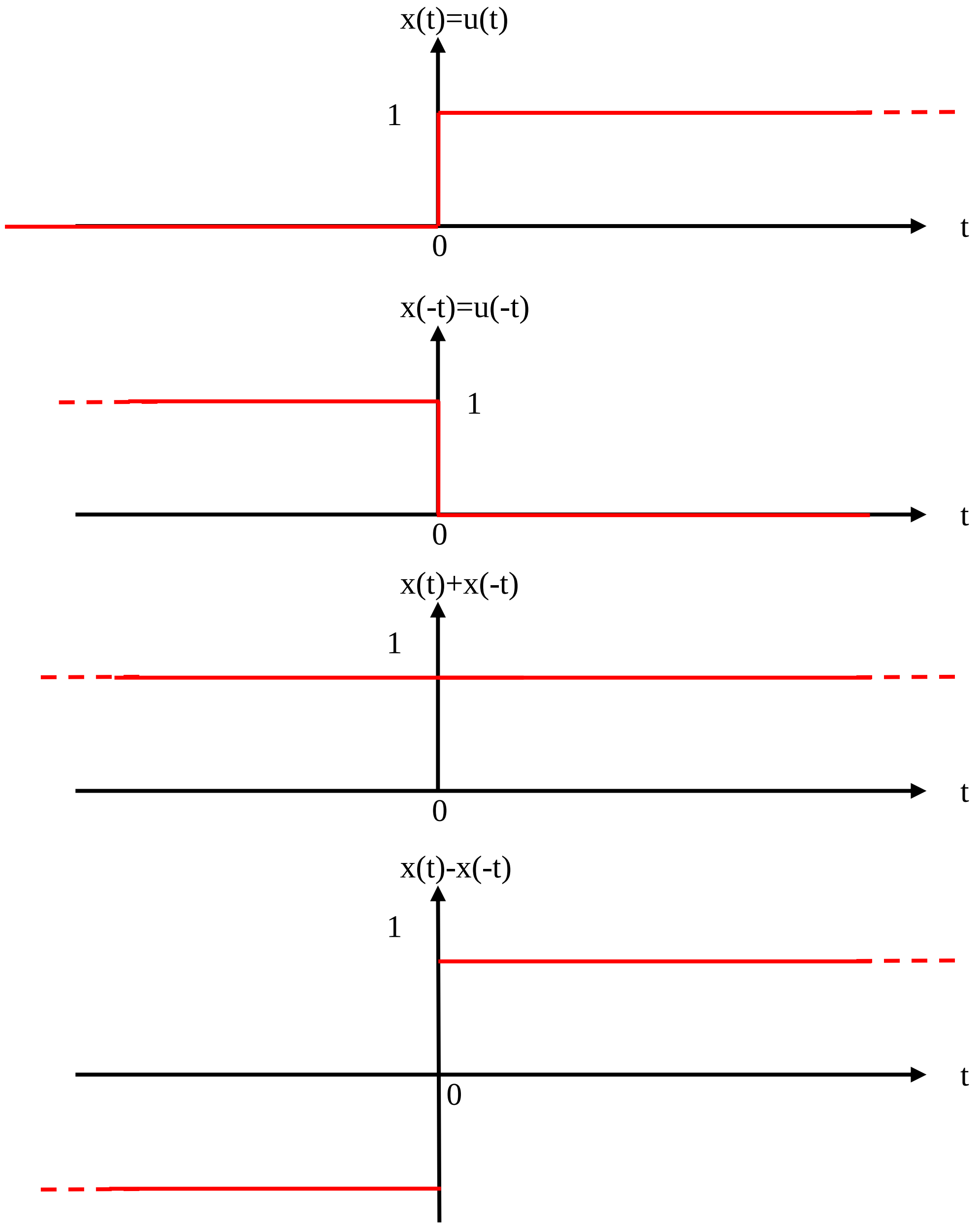


$$\Rightarrow \int_{-\infty}^{\infty} \left|\frac{d^2}{dt^2}x(t)\right| dt = \int_{-\infty}^{\infty} |\delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t-2)| dt$$

$$\begin{aligned} &= \int_{-\infty}^{\infty} |\delta(t+2)| dt + \int_{-\infty}^{\infty} |-\delta(t+1)| dt + \int_{-\infty}^{\infty} |-\delta(t-1)| dt + \int_{-\infty}^{\infty} |\delta(t-2)| dt \\ &= \int_{-\infty}^{\infty} |\delta(t+2)| dt + \int_{-\infty}^{\infty} |\delta(t+1)| dt + \int_{-\infty}^{\infty} |\delta(t-1)| dt + \int_{-\infty}^{\infty} |\delta(t-2)| dt \\ &= 1 + 1 + 1 + 1 \\ &= 4 \end{aligned}$$

(12.4) Test the signal x(t) = u(t) for even and odd.

Given unit step signal, $x(t) = u(t) = \begin{cases} 1 & ; \ t > 0 \\ 0 & ; \ t < 0 \end{cases}$



x(-t) ≠ x(t) ⇒ Given signal is not an even signal &

x(-t) ≠ -x(t) ⇒ Given signal is not a odd signal

therefore, given signal is neither even nor odd signal. Even and odd parts of a signal are

$$x_e(t) = \frac{x(t) + x(-t)}{2} = \frac{u(t) + u(-t)}{2} = \frac{1}{2} = Constant$$

$$x_o(t) = \frac{x(t) - x(-t)}{2} = \frac{u(t) - u(-t)}{2} = \frac{1}{2} Sgn(t)$$

(12.5) Test the signal $y(t) = 3e^{j\frac{\pi}{5}t}$ for even and odd.

Sol: Given signal

$$y(t) = 3e^{j\frac{\pi}{5}t} = 3Cos\left(\frac{\pi}{5}t\right) + j3Sin\left(\frac{\pi}{5}t\right)$$

$$\Rightarrow y(-t) = 3e^{-j\frac{\pi}{5}t} = 3Cos\left(\frac{\pi}{5}t\right) - j3Sin\left(\frac{\pi}{5}t\right)$$

⇒y(-t) ≠ y(t), given signal is not an even signal &

⇒y(-t) ≠ -y(t), given signal is not an odd signal

Therefore, given signal is neither even nor odd signal. Even and odd parts of a signal are

$$y_e(t) = \frac{y(t) + y(-t)}{2} = 3Cos\left(\frac{\pi}{5}t\right)$$

$$y_o(t) = \frac{y(t) - y(-t)}{2} = j3Sin\left(\frac{\pi}{5}t\right)$$

(12.6) Evaluate the conjugate symmetry and anti-symmetry parts of $x(t)=je^{jt}$.

Given, $x(t) = je^{jt} = j(Cost+jSint) = -Sin(t) + jCos(t)$

⇒ x(-t) = Sin(t) + jCos(t) ⇒ x*(-t) = Sin(t) - jCos(t)

Conjugate symmetry part of x(t) is

$$x_{cs}(t) = \frac{x(t)+x^*(-t)}{2} = \frac{-Sin(t)+jCos(t)+Sin(t)-jCos(t)}{2} = 0$$

Conjugate anti-symmetry part of x(t) is

$$x_{cas}(t) = \frac{x(t)-x^*(-t)}{2} = \frac{-Sin(t)+jCos(t)-Sin(t)+jCos(t)}{2} = -Sin(t) + jCos(t) = x(t)$$

Given x(t) is conjugate anti-symmetry signal, because x(t) = -x*(-t).

(12.7) Match the following signals.

a. Unit impulse signal δ(t)	I. Even signal
b. Unit step signal u(t)	II. Odd signal
c. Signum signal sgn(t)	III. Neither even nor odd

(12.8) Test the signal x(t) =$x_1(t)+x_2(t)$ = 2Sin(3t) + 3Cos(4t/3) for periodicity.

If T_1 and T_2 are periods of $x_1(t)$ and $x_2(t)$, then

$T_1 = \frac{2\pi}{3}$ and $T_2 = \frac{2\pi}{\frac{4}{3}} = \frac{3\pi}{2}$

$\Rightarrow \frac{T_1}{T_2} = \frac{2\pi/3}{3\pi/2} = \frac{4}{9} = Rational$

Given signal is periodic with a period of T = $9T_1 = 4T_2 = 6\pi$.

(12.9) Test the signal x(t) =$x_1(t)+x_2(t)$ = 2Sin(3πt) + 3Cos(4πt/3) for periodicity.

If T_1 and T_2 are periods of $x_1(t)$ and $x_2(t)$, then

$T_1 = \frac{2\pi}{3\pi} = \frac{2}{3}$ and $T_2 = \frac{2\pi}{\frac{4\pi}{3}} = \frac{3}{2}$

$\Rightarrow \frac{T_1}{T_2} = \frac{2/3}{3/2} = \frac{4}{9} = Rational$

Given signal is periodic with a period of T = $9T_1 = 4T_2 = 6$.

(12.10) Test the signal x(t) =$x_1(t)+x_2(t)$ = 2Sin(3πt) + 3Cos(4t/3) for periodicity.

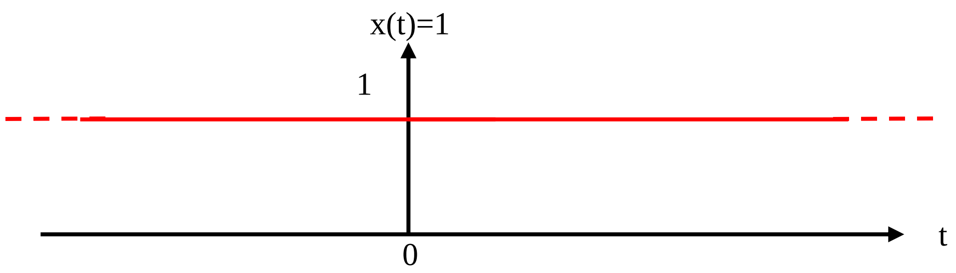
If T_1 and T_2 are periods of $x_1(t)$ and $x_2(t)$, then

$T_1 = \frac{2\pi}{3\pi} = \frac{2}{3}$ and $T_2 = \frac{2\pi}{\frac{4}{3}} = \frac{3\pi}{2}$

$\Rightarrow \frac{T_1}{T_2} = \frac{2/3}{3\pi/2} = \frac{4}{9\pi} = Irrational$

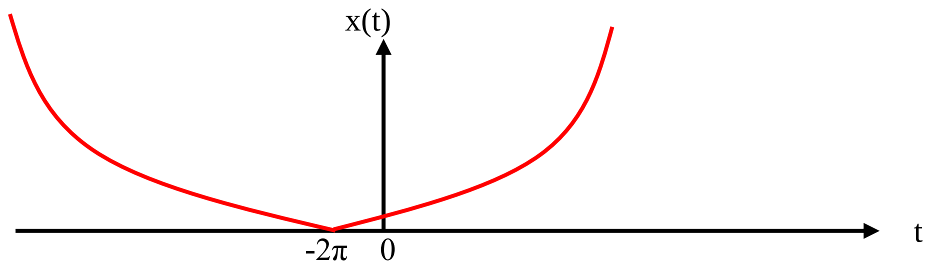
Given signal is aperiodic.

(12.11) Test the signal x(t) = 1 for periodicity.



Given signal is aperiodic.

(12.12) Test the signal x(t) = (t+2π)^2 for periodicity?



Given signal is aperiodic.

(12.13) Test the signal $x(t) = e^{j2\pi f_0 t}, f_0 = log(3)$ for periodicity?

$Given, x(t) = e^{j2\pi t \, log(3)} = e^{log\, 3^{j2\pi t}} = 3^{j2\pi t}$

$e^{j2\pi t}$ is periodic, but $3^{j2\pi t}$ *is aperiodic*

Therefore, given signal is aperiodic.

(12.14) Test the signal $x(t) = e^{j2\pi f_0 t}, f_0 = 3$ for periodicity?

Sol: $x(t) = e^{j2\pi f_0 t} = e^{j2\pi 3t} = e^{j6\pi t}$

$e^{j6\pi t}$ is periodic with a period, $T = \frac{2\pi}{6\pi} = \frac{1}{3}$

(12.15) Test the signal x(t) = t for periodicity?

Given signal is aperiodic and it is a ramp signal.

(12.16) Test the signal x(t) for Energy and Power by Evaluating the Total Energy and the Average Power, given x(t)=e^{-at}u(t), a>0.

Total Energy (E):

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |e^{-at}u(t)|^2 dt$$

$$= \int_{0}^{\infty} e^{-2at} dt$$

$$= \frac{e^{-2at}}{-2a}\Bigg|_{0}^{\infty}$$

$$= \frac{0-1}{-2a}$$

$$= \frac{1}{2a}$$

Average Power (P):

$$P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |e^{-at}u(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{0}^{T} e^{-2at} dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \frac{e^{-2at}}{-2a}\Bigg|_{0}^{T}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \frac{e^{-2aT}-1}{-2a}$$

$$= \underset{T\to\infty}{Lt} \frac{1-e^{-2aT}}{4aT}$$

$$= \frac{1-0}{\infty}$$

$$= 0$$

Given x(t) is **Energy Signal**, because it has finite energy($0<E<\infty$) and zero average power(P=0)

(12.17) Test the signal x(t) for Energy and Power by Evaluating the Total Energy and the Average Power, given x(t)=u(t).

Total Energy (E):

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |u(t)|^2 dt$$

$$= \int_{0}^{\infty} dt$$

$$= t \Big|_{0}^{\infty}$$

$$= \infty - 0$$

$$= \infty$$

Average Power (P):

$$P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |u(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{0}^{T} dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} t \Big|_{0}^{T}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} (T - 0)$$

$$= \underset{T\to\infty}{Lt} \frac{T}{2T}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2}$$

$$= \frac{1}{2}$$

Given x(t) is **Power Signal**, because it has finite average power ($0<P<\infty$) and infinite energy ($E=\infty$).

(12.18) Test the signal x(t) for Energy and Power by Evaluating the Total Energy and the Average Power, given x(t)=e^{at}u(t), a>0.

Total Energy (E):

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |e^{at}u(t)|^2 dt$$

$$= \int_{0}^{\infty} e^{2at} dt$$

$$= \frac{e^{2at}}{2a}\Big|_0^{\infty}$$

$$= \frac{\infty - 1}{2a}$$

$$= \infty$$

Average Power (P):

$$P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |e^{at}u(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{0}^{T} e^{2at} dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \frac{e^{2at}}{2a}\Big|_0^{T}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \frac{e^{2aT} - 1}{2a}$$

$$= \underset{T\to\infty}{Lt} \frac{2ae^{2aT}}{4a}$$

$$= \underset{T\to\infty}{Lt} \frac{e^{2aT}}{2}$$

$$= \infty$$

For an **Energy Signal**, the total energy is finite(0<E<∞) and average power is zero (P=0)
For a **Power Signal**, the average power is finite(0<P<∞) and total energy is infity (E=∞)
It fails to satisfy above two properties, hence the signal x(t) is **neither Energy nor Power Signal**.

(12.19) Test the signal x(t) for Energy and Power by Evaluating the Total Energy and the Average Power, given x(t)=ACos(wt).

Total Energy, $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$= \int_{-\infty}^{\infty} |ACos(wt)|^2 dt$$

$$= A^2 \int_{-\infty}^{\infty} Cos^2(wt) dt$$

$$= \frac{A^2}{2} \int_{-\infty}^{\infty} (1 + Cos(2wt)) dt$$

$$= \frac{A^2}{2} \left(t \Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} Cos(2wt) dt \right)$$

$$= \frac{A^2}{2} (\infty + 0)$$

$$= \infty$$

Average Power, $P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |ACos(wt)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{2T} \int_{-T}^{T} Cos^2(wt) dt$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{4T} \int_{-T}^{T} (1 + Cos(2wt)) dt$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{4T} \left(t \Big|_{-T}^{T} + \int_{-T}^{T} Cos(2wt) dt \right)$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{4T} (2T + 0)$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{2}$$

$$= \frac{A^2}{2}$$

Given x(t) is **Power Signal**, because it has finite average power ($0<P<\infty$) and infinite energy ($E=\infty$).

(12.20) Test the signal x(t) for Energy and Power by Evaluating the Total Energy and the Average Power, given x(t)=ASin(wt).

Total Energy, $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$= \int_{-\infty}^{\infty} |ASin(wt)|^2 dt$$

$$= A^2 \int_{-\infty}^{\infty} Sin^2(wt) dt$$

$$= \frac{A^2}{2} \int_{-\infty}^{\infty} (1 - Cos(2wt)) dt$$

$$= \frac{A^2}{2} \left(t \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} Cos(2wt) dt \right)$$

$$= \frac{A^2}{2} (\infty - 0)$$

$$= \infty$$

Average Power, $P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |ASin(wt)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{2T} \int_{-T}^{T} Sin^2(wt) dt$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{4T} \int_{-T}^{T} (1 - Cos(2wt)) dt$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{4T} \left(t \Big|_{-T}^{T} - \int_{-T}^{T} Cos(2wt) dt \right)$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{4T} (2T - 0)$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{2}$$

$$= \frac{A^2}{2}$$

Given x(t) is **Power Signal**, because it has finite average power (0<P<∞) and infinite energy (E=∞).

(12.21) Test the sinusoidal signals x(t) = Sin(mt) and g(t) = Sin(nt) for orthogonality over the interval (0,2π) for integer values m and n, m ≠ n.

Evaluate the Integral $\int_{t_1}^{t_2} x(t)g(t)dt$, if it is zero, then the signals x(t) & g(t) are orthogonal

$$= \int_0^{2\pi} Sin(mt)Sin(nt)dt$$

$$= \frac{1}{2}\int_0^{2\pi} \big(Cos(mt - nt) - Cos(mt + nt)\big)dt$$

$$= \frac{1}{2}\int_0^{2\pi} (Cos(m - n)t - Cos(m + n)t)dt$$

$$= \frac{1}{2}\left(\frac{Sin(m - n)t}{m - n} - \frac{Sin(m + n)t}{m + n}\right)\Big|_0^{2\pi}$$

$$= \frac{1}{2}\left(\frac{Sin(m - n)2\pi - Sin(0)}{m - n} - \frac{Sin(m + n)2\pi - Sin(0)}{m + n}\right)$$

$$= \frac{1}{2}\left(\frac{0 - 0}{m - n} - \frac{0 - 0}{m + n}\right) = 0$$

Given signals x(t) = Sin(mt) and g(t) = Sin(nt) are orthogonal.

(12.22) Examine the complex signals x(t) = e^{jnwot} and g(t) = e^{jmwot} for orthogonality over the interval (0,2π) for integer values m ≠ n

Evaluate the Integral $\int_{t_1}^{t_2} x(t)g^*(t)dt$, if it is zero, then x(t) & g(t) are orthogonal.

$$= \int_0^{2\pi} e^{jnw_0t}\left(e^{jmw_0t}\right)^* dt$$

$$= \int_0^{2\pi} e^{jnw_0t}e^{-jmw_0t}dt$$

$$= \int_0^{2\pi} e^{j(n-m)w_0t}dt$$

$$= \frac{e^{j(n-m)w_0t}}{j(n-m)w_0}\Big|_0^{2\pi}$$

$$= \frac{e^{j(n-m)w_02\pi} - 1}{j(n-m)w_0}$$

$$= \frac{1 - 1}{j(n-m)w_0}$$

$$= 0$$

Given complex signals are orthogonal.

(12.23) A rectangular signal x(t) is defined by

$$\boldsymbol{x(t)} = \begin{cases} \mathbf{1}, & \mathbf{0 < t < \pi} \\ \mathbf{-1}, & \mathbf{\pi < t < 2\pi} \end{cases}$$

(a) Approximate the above rectangular signal x(t) by using a sinusoid signal g(t)=Sin(t) over the interval (0,2π).

(b) Compute the mean square error in the approximation process.

(a) A real valued signal x(t) over the interval (t_1, t_2) can be approximated by another signal g(t) over the same interval (t_1, t_2) is x(t) = C g(t), where, C is the approximation constant.

$$C = \frac{\int_{t_1}^{t_2} x(t)g(t)dt}{\int_{t_1}^{t_2} g^2(t)dt} = \frac{\int_{t_1}^{t_2} x(t)g(t)dt}{K}$$

$$K = \int_{t_1}^{t_2} g^2(t)dt$$

$$= \int_0^{2\pi} Sin^2(t)dt$$

$$= \int_0^{2\pi} \left(\frac{1 - Cos(2t)}{2}\right) dt$$

$$= \frac{1}{2}\left(t - \frac{Sin(2t)}{2}\right) \Big|_0^{2\pi}$$

$$= \frac{1}{2}(2\pi - 0)$$

$$= \pi$$

$$C = \frac{\int_{t_1}^{t_2} x(t)g(t)dt}{K}$$

$$= \frac{1}{\pi}\int_0^{2\pi} x(t)Sin(t)dt$$

$$= \frac{1}{\pi}\left(\int_0^{\pi} Sin(t)dt - \int_{\pi}^{2\pi} Sin(t)dt\right)$$

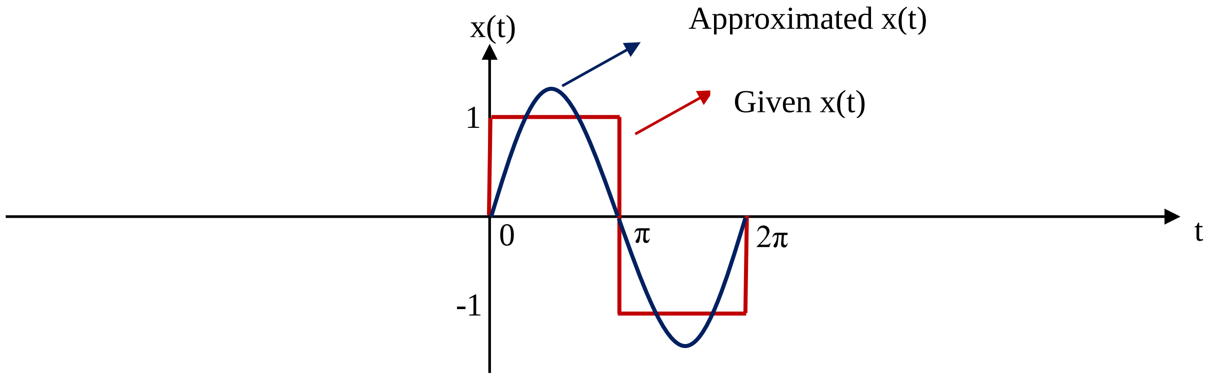
$$= \frac{1}{\pi}\left(-Cos(t)\Big|_0^{\pi} + Cos(t)\Big|_{\pi}^{2\pi}\right)$$

$$= \frac{1}{\pi}(-(-1) + 1 + 1 - (-1))$$

$$= \frac{4}{\pi}$$

Approximated signal,

$$x(t) = \frac{4}{\pi} Sin(t)$$



(b)Mean Square Error

$$\epsilon = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt - C^2K\right)$$

$$= \frac{1}{2\pi - 0}\left(\int_0^{2\pi} x^2(t)dt - \left(\frac{4}{\pi}\right)^2 \pi\right)$$

$$= \frac{1}{2\pi}\left(\int_0^{\pi} (1)^2 dt + \int_{\pi}^{2\pi} (-1)^2 dt - \frac{16}{\pi^2}\pi\right)$$

$$= \frac{1}{2\pi}\left(\int_0^{\pi} dt + \int_{\pi}^{2\pi} dt - \frac{16}{\pi}\right)$$

$$= \frac{1}{2\pi}\left(t\Big|_0^{\pi} + t\Big|_{\pi}^{2\pi} - \frac{16}{\pi}\right)$$

$$= \frac{1}{2\pi}\left(\pi - 0 + 2\pi - \pi - \frac{16}{\pi}\right)$$

$$= \frac{1}{2\pi}\left(2\pi - \frac{16}{\pi}\right)$$

$$= 1 - \frac{8}{\pi^2}$$

$$= 0.189 \;\; or \; 18.9\%$$

(12.24) A rectangular signal x(t) is defined by

$$x(t) = \begin{cases} 1, & 0 < t < \pi \\ -1, & \pi < t < 2\pi \end{cases}$$

(a) Approximate the above rectangular signal x(t) by using a set of sinusoid signal g(t)=Sin(t), Sin(2t), Sin(3t), ….. over the interval (0,2π).

(b) Compute the mean square error in the approximation process.

(a) We know the approximation of given x(t) by a set of mutually orthogonal signals $g_1(t)$, $g_2(t)$, ……… $g_n(t)$ over the same interval (t_1, t_2) is

$x(t) = C_1 g_1(t) + C_2 g_2(t) + \dots\dots\dots C_r g_r(t) + \dots\dots\dots\dots\dots C_n g_n(t)$

$$x(t) = \sum_{r=1}^{n} C_r g_r(t)$$

Where, $g_r(t) = \text{Sin}(r\,t)$.

$$C_r = \frac{\int_{t_1}^{t_2} x(t) g_r(t) dt}{\int_{t_1}^{t_2} g_r^{\,2}(t) dt} = \frac{\int_{t_1}^{t_2} x(t) g_r(t) dt}{k_r}$$

$$k_r = \int_{t_1}^{t_2} g_r^{\,2}(t) dt$$

$$K_r = \int_0^{2\pi} Sin^2(rt) dt$$

$$= \int_0^{2\pi} \left(\frac{1 - Cos(2rt)}{2}\right) dt$$

$$= \frac{1}{2}\left(t - \frac{Sin(2rt)}{2r}\right)\Big|_0^{2\pi}$$

$$= \frac{1}{2}(2\pi - 0)$$

$$K_r = \pi$$

$$C_r = \frac{\int_{t_1}^{t_2} x(t)g_r(t)dt}{K_r}$$

$$= \frac{1}{\pi}\int_0^{2\pi} x(t)Sin(rt)dt$$

$$= \frac{1}{\pi}\left(\int_0^{\pi} Sin(rt)dt - \int_{\pi}^{2\pi} Sin(rt)dt\right)$$

$$= \frac{1}{r\pi}\left(-Cos(rt)\Big|_0^{\pi} + Cos(rt)\Big|_{\pi}^{2\pi}\right)$$

$$= \frac{1}{r\pi}(-Cos(r\pi) + 1 + 1 - Cos(r\pi))$$

$$= \frac{2}{r\pi}(1 - Cos(r\pi))$$

$$= \frac{2}{r\pi}(1 - (-1)^r)$$

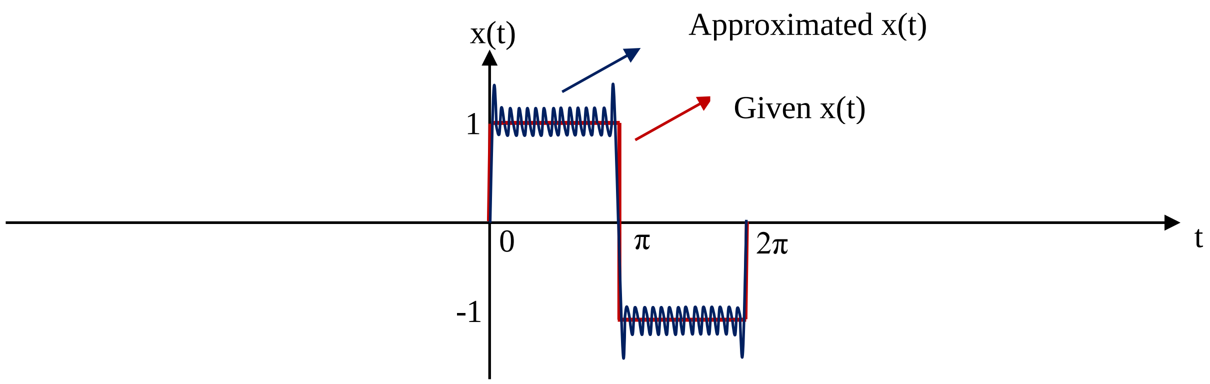
$$C_r = \begin{cases} \frac{4}{r\pi}, & if\ r: odd \\ 0, & if\ r: even \end{cases}$$

$$C_1 = \frac{4}{\pi}, C_3 = \frac{4}{3\pi}, C_5 = \frac{4}{5\pi}, C_7 = \frac{4}{7\pi}, C_9 = \frac{4}{9\pi}, \ldots \ldots \ldots \ldots \ldots,$$

$$C_2 = C_4 = C_6 = C_8 = \cdots = 0$$

Approximated signal,

$$x(t) = \tfrac{4}{\pi}Sin(t) + \tfrac{4}{3\pi}Sin(3t) + \tfrac{4}{5\pi}Sin(5t) + \tfrac{4}{7\pi}Sin(7t) + \cdots$$



(b) Mean Square Error

$$\epsilon_r = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt - \sum_{r=1}^{n} C_r{}^2 K_r\right)$$

$$= \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} x^2(t)dt - \left(C_1{}^2K_1 + C_2{}^2K_2 + C_3{}^2K_3 + \cdots\right)\right)$$

$$= \frac{1}{2\pi - 0}\left(\int_0^{2\pi} x^2(t)dt - \left(\left(\frac{4}{\pi}\right)^2 \pi + \left(\frac{4}{3\pi}\right)^2 \pi + \left(\frac{4}{5\pi}\right)^2 \pi + \cdots\right)\right)$$

$$= \frac{1}{2\pi}\left(\int_0^{\pi} (1)^2 dt + \int_{\pi}^{2\pi} (-1)^2 dt - \left(\frac{16}{\pi^2}\pi + \frac{16}{9\pi^2}\pi + \frac{16}{25\pi^2}\pi + \cdots\right)\right)$$

$$= \frac{1}{2\pi}\left(\int_0^{\pi} dt + \int_{\pi}^{2\pi} dt - \left(\frac{16}{\pi} + \frac{16}{9\pi} + \frac{16}{25\pi} + \cdots\right)\right)$$

$$= \frac{1}{2\pi}\left(t\Big|_0^{\pi} + t\Big|_{\pi}^{2\pi} - \frac{16}{\pi}\left(1 + \frac{1}{9} + \frac{1}{25} + \cdots\right)\right)$$

$$= \frac{1}{2\pi}\left(\pi - 0 + 2\pi - \pi - \frac{16}{\pi}\left(1 + \frac{1}{9} + \frac{1}{25} + \cdots\right)\right)$$

$$= \frac{1}{2\pi}\left(2\pi - \frac{16}{\pi}\left(1 + \frac{1}{9} + \frac{1}{25} + \cdots\right)\right)$$

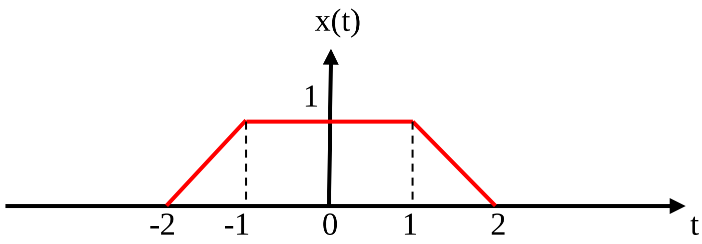
$$= 1 - \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25} + \cdots\right)$$

$= 0$, if we consider more number of terms

13. Assignment Questions:

(1) Evaluate the average power of a signal $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, given that x(t) = Sin(t) u(t).

(2) Draw the graphical representation of signal y(t) = 2 x(-3t+4) by using operations



(3) Test the signal x(t)= 5 Sin^4(t)+3 Sin^2(t) for periodicity. If it is periodic, then find the period.

(4) Draw the graphical representation of signals u(t – 2), u(t + 2), u(-t+2) and u(-t-2).

(5) Draw the graphs of signals x(2t) and x(t/2), if x(t) = 3Cos(πt).

(6) Why the continuous time signal x(t) = 2 Cos(3t) + 3 Cos(πt) is aperiodic and non-causal? Justify.

(7) Obtain the period of a periodic signal x(t) = 2Sin(t) + 3Cos(2t) + 4Sin(3t)

(8) Compute and plot the convoluted signal x(t) * y(t), given x(t) = u(t-1) – u(t-4), y(t) = e^{-2t}u(t)

(9) Obtain the convolution of the signals x(t) = u(t) and y(t) = u(t) - u(t-2)

(10)Test the signals for orthogonality over the interval (0,2π) for integer values of m and n, m≠n.

(a) x(t) = Cos(mt) and g(t) = Cos(nt)

(b) x(t) = Sin(mt) and g(t) = Cos(nt)

(c) x(t) = Cos(mt) and g(t) = Sin(nt)

(11)Examine the complex signals x(t) = e^{jnwot} and g(t) = e^{-jmwot} for orthogonality over the interval (0,2π) for integer values of m and n, m ≠ n.

14. Quiz Questions:

(1) The image captured by a cell phone is an example of

(a) Discrete time signal

(b) 2D signal

(c) Energy signal

(d) All the above

(2) The signal $e^{-\frac{1}{2}|t|}$ is

(a) Power signal with power P = 2

(b) Power signal with power P = 1

(c) Energy signal with energy E = 2

(d) Energy signal with energy E = 1

(3) Total energy under a rectangular signal x(t) = Arect(t/T) is

(a) AT

(b) A^2T

(c) A^2T^2

(d) None

(4) What is the energy of the signal y(t) = x(3t), if the energy of the signal x(t) is E.

(a) E

(b) 3E

(c) 9E

(d) E/3

(5) Average power of the signal x(t) = 8Cos(20πt-π/2)+4Sin(15πt).

(a) 40

(b) 41

(c) 42

(d) 82

x(t)= 8Cos(20πt-π/2)+4Sin(15πt)

x(t)= 8Sin(20πt)+4Sin(15πt)

$$P_{avg} = \frac{8^2+4^2}{2} = \frac{64+16}{2} = 40$$

(6) What is the energy of the signal y(t) = 2x(3t), if the energy of the signal x(t) is E.

(a) 6E

(b) 12E

(c) 36E

(d) 4E/3

(7) The signal δ(-2t) is equals to

(a) $\boldsymbol{\delta}\left(\frac{t}{2}\right)$

(b) $\frac{1}{2}\boldsymbol{\delta}(\boldsymbol{t})$

(c) $-\frac{1}{2}\delta(t)$

(d) $-2\delta(t)$

(8) Shifting property of the impulse signal is

(a) $\int_{-\infty}^{\infty} \boldsymbol{x}(\boldsymbol{t})\boldsymbol{\delta}(\boldsymbol{t})\boldsymbol{dt} = \boldsymbol{x}(\mathbf{0})$

(b) $\boldsymbol{x}(\boldsymbol{t})\boldsymbol{\delta t}) = \boldsymbol{x}(\mathbf{0})\boldsymbol{\delta}(\mathbf{0})$

(c) $\int_{-\infty}^{\infty} \boldsymbol{x}(\boldsymbol{\tau})\boldsymbol{\delta}(\boldsymbol{t}-\boldsymbol{\tau})\boldsymbol{d\tau} = \boldsymbol{x}(\boldsymbol{t})$

(d) $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$

(9) What is the convolution of two rectangular signals having same width?

(a) Trapezoidal signal

(b) Triangular signal

(c) Rectangular signal

(d) Sinc signal

(10) What is the convolution of two rectangular signals with unequal widths?

(a) Trapezoidal signal

(b) Triangular signal

(c) Rectangular signal

(d) Sinc signal

(11) The value of integral $\int_{-\infty}^{\infty} \boldsymbol{\delta}(\boldsymbol{t})\, \boldsymbol{cos}(\,\mathbf{3}\boldsymbol{t}/\mathbf{2})\boldsymbol{dt}$ **is**

(a) Zero

(b) One

(c) Infinity

(d) Undefined

(12) Double integration of a unit step signal is

(a) An impulse

(b) A parabola

(c) A ramp

(d) A doublet

(13) Double differentiation of a unit step or differentiation of unit impulse signal is

(a) An impulse

(b) A parabola

(c) A ramp

(d) A doublet

(14)The value of doublet signal at t = 0 is

(a) 0

(b) ∞

(c) $-\infty$

(d) (b) and (c)

(15)Which of the following signal is conjugate symmetry.

(a) x(t) = Cos(t) + jSin(t)

(b) y(t) = Sin(t) + jCos(t)

(c) $z(t) = Cos(t) + jt^2$

(d) w(t) = Sin(t) + jt

x(-t) = Cos(t) - jSin(t) $\Rightarrow$ x*(-t) = Cos(t) + jSin(t) $\Rightarrow$ x*(-t) = x(t)

(16)Which of the following signal is conjugate anti-symmetry.

(a) x(t) = Cos(t) + jSin(t)

(b) y(t) = Sin(t) + jCos(t)

(c) $z(t) = Cos(t) + jt^2$

(d) w(t) = Sin(t) + jt

y(-t) = -Sin(t) + jCos(t) $\Rightarrow$ y*(-t) = -Sin(t) - jCos(t) $\Rightarrow$ y*(-t) = -y(t)

(17) Which of the following signals are neither conjugate symmetry nor conjugate anti-symmetry

(a) x(t) = Cos(t) + jSin(t)

(b) y(t) = Sin(t) + jCos(t)

(c) z(t) = Cos(t) + jt^2

(d) w(t) = Sin(t) + jt

z(-t) = Cos(t) + jt^2 ⇒ z*(-t) = Cos(t) - jt^2 ⇒ z*(-t) ≠ ±z(t)

w(-t) = -Sin(t) - jt ⇒ w*(-t) = -Sin(t) + jt ⇒ w*(-t) ≠ ±w(t)

(18) Conjugate symmetry and anti-symmetry parts of x(t) = j + t.

(a) Cos(t), jSin(t)

(b) Sin(t), jCos(t)

(c) jCos(t), -Sin(t)

(d) None

x(t) = j + t ⇒ x(-t) = j – t ⇒ x*(-t) = - j - t

$$x_{cs}(t) = \frac{x(t) + x*(-t)}{2} = \frac{j + t - j - t}{2} = 0$$

$$x_{cas}(t) = \frac{x(t) - x*(-t)}{2} = \frac{j + t + j + t}{2} = j + t$$

(19) Even and odd parts of unit step signal u(t) is

(a) ½, ½Sgn(t)

(b) 1, Sgn(t)

(c) 1, 0

(d) ½, -½Sgn(t)

$$u(t) = \frac{1 + Sgn(t)}{2} = \frac{1}{2} + \frac{Sgn(t)}{2}$$

(20) The dirac delta signal is defined as

(a) $\delta(t) = \begin{cases} 1 & ; \ t = 0 \\ 0 & ; \ t \neq 0 \end{cases}$

(b) $\delta(t) = \begin{cases} \infty & ; \ t = 0 \\ 0 & ; \ t \neq 0 \end{cases}$

(c) $\delta(t) = \begin{cases} 1 & ; \ t = 0 \\ 0 & ; \ t \neq 0 \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

(d) $\boldsymbol{\delta(t) = \begin{cases} \infty & ; \ t = 0 \\ 0 & ; \ t \neq 0 \end{cases}}$ **and** $\boldsymbol{\int_{-\infty}^{\infty} \delta(t)dt = 1}$

(21) What is the fundamental frequency of a signal

x(t) = 30Sin(100t) + 100Cos(300t) + 6Sin(500t) in rad/sec.

(a) 100

(b) 300

(c) 500

(d) 1500

(22) Choose even signals from (i)δ(t) (ii)u(t) (iii)sgn(t) (iv)Arect(t/T) (v)ASinc(t) (vi)floot(t) (vii)e^{-at}u(t) (viii)e^{at}u(-t) (ix)$e^{-a|t|}$ (x)$e^{a|t|}$ (xi)e^{-at} (xii) e^{at} (xiii)r(t)=tu(t), assume a>0.

Ans: (i), (iv), (v), (ix), (x)

(23) Choose odd signals from (i)δ(t) (ii)u(t) (iii)sgn(t) (iv)Arect(t/T) (v)ASinc(t) (vi)floot(t) (vii)e^{-at}u(t) (viii)e^{at}u(-t) (ix)$e^{-a|t|}$ (x)$e^{a|t|}$ (xi)e^{-at} (xii) e^{at} (xiii)r(t)=tu(t), assume a>0.

Ans: (iii)

(24) Choose neither even nor odd signals from (i)δ(t) (ii)u(t) (iii)sgn(t) (iv)Arect(t/T) (v)ASinc(t) (vi)floot(t) (vii)e^{-at}u(t) (viii)e^{at}u(-t) (ix)$e^{-a|t|}$ (x)$e^{a|t|}$ (xi)e^{-at} (xii) e^{at} (xiii)r(t)=tu(t), assume a>0.

Ans: (ii), (vi), (vii), (viii), (xi), (xii), (xiii)

(25) Choose causal signals from (i)δ(t) (ii)u(t) (iii)sgn(t) (iv)Arect(t/T) (v)ASinc(t) (vi)floot(t) (vii)e^{-at}u(t) (viii)e^{at}u(-t) (ix)$e^{-a|t|}$ (x)$e^{a|t|}$ (xi)e^{-at} (xii) e^{at} (xiii)r(t)=tu(t), assume a>0.

Ans: (i), (ii), (vii), (xiii)

(26) Choose anti-causal signals from (i)δ(t) (ii)u(t) (iii)sgn(t) (iv)Arect(t/T) (v)ASinc(t) (vi)floot(t) (vii)e^{-at}u(t) (viii)e^{at}u(-t) (ix)$e^{-a|t|}$ (x)$e^{a|t|}$ (xi)e^{-at} (xii) e^{at} (xiii)r(t)=tu(t), assume a>0.

Ans: (viii)

(27) Choose non-causal signals from (i)δ(t) (ii)u(t) (iii)sgn(t) (iv)Arect(t/T) (v)ASinc(t) (vi)floot(t) (vii)e^{-at}u(t) (viii)e^{at}u(-t) (ix)$e^{-a|t|}$ (x)$e^{a|t|}$ (xi)e^{-at} (xii) e^{at} (xiii)r(t)=tu(t), assume a>0.

Ans: (iii), (iv), (v), (vi), (viii), (ix), (x), (xi), (xii)

(28) Choose bounded signals from (i)δ(t) (ii)u(t) (iii)sgn(t) (iv)Arect(t/T) (v)ASinc(t) (vi)floot(t) (vii)e^{-at}u(t) (viii)e^{at}u(-t) (ix)$e^{-a|t|}$ (x)$e^{a|t|}$ (xi)e^{-at} (xii) e^{at} (xiii)r(t)=tu(t), assume a>0.

Ans: (ii), (iii), (iv), (v), (vii), (viii), (ix)

(29) Choose unbounded signals from (i)δ(t) (ii)u(t) (iii)sgn(t) (iv)Arect(t/T) (v)ASinc(t) (vi)floot(t) (vii)e^{-at}u(t) (viii)e^{at}u(-t) (ix)$e^{-a|t|}$ (x)$e^{a|t|}$ (xi)e^{-at} (xii) e^{at} (xiii)r(t)=tu(t), assume a>0.

Ans: (i), (vi), (x), (xi), (xii), (xiii)

(30) Choose periodic signals from (i)δ(t) (ii)u(t) (iii)sgn(t) (iv)Arect(t/T) (v)ASinc(t) (vi)floot(t) (vii)e^{-at}u(t) (viii)e^{at}u(-t) (ix)$e^{-a|t|}$ (x)$e^{a|t|}$ (xi)e^{-at} (xii) e^{at} (xiii)r(t)=tu(t), assume a>0.

Ans: Nil

(31) Choose aperiodic signals from (i)δ(t) (ii)u(t) (iii)sgn(t) (iv)Arect(t/T) (v)ASinc(t) (vi)floot(t) (vii)e^{-at}u(t) (viii)e^{at}u(-t) (ix)$e^{-a|t|}$ (x)$e^{a|t|}$ (xi)e^{-at} (xii) e^{at} (xiii)r(t)=tu(t), assume a>0.

Ans: All

(32) Which of the following signal involves time and amplitude scaling operations

(a) 2x(3t)

(b) x(3t+2)

(c) 2x(t+3)

(d) x(-3t+2)

(33) Which of the following signal involves time reversal and time shifting operations

(a) -x(t-2)

(b) 2x(-t)

(c) -x(t+2)

(d) x(-t+2)

(34) Which of the following signal involves time scaling, time shifting and time reversal operations

(a) -2x(3t-4)

(b) x(-3t+2)

(c) x(-t+2)

(d) -x(2t+3)

(35) Which of the following signal involves time scaling, time shifting and time reversal and amplitude scaling operations

(a) -x(3t-4)

(b) x(-3t+4)

(c) 2x(-3t+4)

(d) 2x(3t-4)

(36) If x(t) and g(t) are orthogonal, then the approximation of x(t) by using g(t) is

(A) $\boldsymbol{x(t) = Cg(t)}$

(B) $\boldsymbol{x(t) = C\int_{-\infty}^{\infty} g(t)\,dt}$

(C) $x(t) = C\frac{d}{dt}g(t)$

(D) Approximation is not possible

(37) What is the condition to be satisfied for Orthogonality between x(t) and g(t)?

(A) $\boldsymbol{\int_{-\infty}^{\infty} x(t)g(t)dt = 0}$

(B) $\boldsymbol{\int_{-\infty}^{\infty} g(t)\,dt = 0}$

(C) $\int_{-\infty}^{\infty} x(t)\,dt = 0$

(D) $\frac{d}{dt}(x(t)g(t)) = 0$

(38) If the approximation is x(t) = C g(t), then what is the expression for a constant C?

(A) $\boldsymbol{C = \frac{\int_{t_1}^{t_2} x(t)g(t)dt}{\int_{t_1}^{t_2} g(t)dt}}$

(B) $\boldsymbol{C = \frac{\int_{t_1}^{t_2} x(t)g(t)dt}{\int_{t_1}^{t_2} g^2(t)dt}}$

(C) $C = \frac{\int_{t_1}^{t_2} x(t)g(t)dt}{\int_{t_1}^{t_2} x(t)dt}$

(D) $C = \frac{\int_{t_1}^{t_2} x(t)g(t)dt}{\int_{t_1}^{t_2} x^2(t)dt}$

(39) Which of the following signals are orthogonal?

(A) Sin(t) and Sin(2t)

(B) Sin(t) and Cos(t)

(C) Cos(t) and Cos(2t)

(D) All the above

(40) If x(t) is approximated with more number of orthogonal signals, then the mean square error

(A) Increases

(B) Constant

(C) Decreases

(D) None of the above